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COMPUTER SIMULATION OF CERTAIN STOCHASTIC

RELATIONSHIPS IN MICRO-ECONOMIC SYSTEMS

A DISSERTATION

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CHAPTER I

INTRODUCTION

In studying any economic system, one would like generally to be able to describe the system, to identify and relate its elementary parts, and then once given the system's input, to predict its output. The description can be either verbal or mathematical. But, on the one hand, verbal description is not exact and often needs quantification, and on the other, it may be impossible to describe by mathematical equations the complicated and dynamic relationships between the parts of the system. Moreover, assuming the equation set is adequate for the purpose, a solution of the equations may not exist. Since the invention of the electronic computer, however, a research technique new to the social science, called "simulation," has been developed to aid in explaining and predicting the behavior of complex economic systems.

The term "simulation" as used here is in accordance with John Harling's definition.[1]

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[1]John Harling, "Simulation Techniques in Operations Research - A Review," Operation Research, Volume VI (May-June, 1958), p. 307.

By simulation is meant the technique of setting up a stochastic model of a real situation, and then performing sampling experiments upon the model. The feature which distinguishes a simulation from a mere sampling experiment in the classical sense is that of the stochastic model. Whereas a classical sampling experiment in statistics is most often performed directly upon raw data, a simulation entails first of all the construction of an abstract model of the system to be studied.

Early use of simulation involved experimenting with models given a physical embodiment.[2] For example, a sand table was used by the military to represent a small scale model of actual geographical terrain. Social phenomena, however, are not easily given simple physical embodiment, thus limiting the use of this type of simulation approach in the field of economics. In fact, only few attempts have met with any success, a famous example being the hydraulic model built by Irving Fisher[3] at the London School of Economics to help explain the circular flows of money and commodities in a macro-economic system.

The advent of the electronic digital computer opens a new way for simulating economic systems without relying on a physical embodiment. In essence, the complicated

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[2] G. H. Orcutt, "Simulation of Economic Systems," The American Economic Review, Vol. D (Dec. 1960), p. 895.

[3] I. Fisher, Mathematical Investigations in the Theory of Value and Prices (New Haven, 1925), p. 44. This model was built basically as a teaching device.

relationships of an economic system are simulated through the use of the logic-arithmetic device of the computer. The following is a review of the literature dealing with computer simulations in the field of economics.

Literature Review

There has been only one attempt to simulate a national macrotype model; namely, a demographic model of the United States household sector, with individual human beings as its basic components, by Orcutt, Greenberger, Korbel and Rivlin.[4] An initial simulated population of over 10,000 individuals was made to be representative of the United States population as of April, 1950. Each individual in the population is distinguished by his respective age, sex, race, and marital status, each month constituting the time interval for a simulation run. Random drawings from specified probability distributions are used to determine for each simulation run the possible changes in population characteristics such as births, deaths, marriages, or divorces of the individuals.

In the field of micro-economics, many simulation models have been constructed. For convenience of

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[4]G. H. Orcutt, and Associates, Micro-analysis of Social Economic Systems (New York: Harper, 1961).

presentation, these models have been divided into three groups. The first group is concerned with the internal mechanism of a firm. One example is F. M. Tonge's heuristic model[5] that balances an assembly line in a factory. His objective was to find the minimum number of workmen consistent with a given rate of product assembly. Problems such as job shop scheduling, and personnel and equipment assignment, which customarily are too large to be solved by usual analytical methods, have been treated simultaneously. By trial and error, Tonge has succeeded in finding efficient ways of assigning the elemental tasks making up the assembly operation to work stations along the assembly line.

The second group of micro-economic simulation models deals with industries in an attempt to analyze the interrelationships of firms and their environment. One example is Cyert, Feigenbaum and March's duopoly model.[6] The basic assumption in their model is that firms will set their output and price for the subsequent period by forecasting the reactions of their competitors and revising their estimates of market demand and their own cost curves each period in order

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[5] F. M. Tonge, "A Heuristic Program for Assembly Line Balancing," Thesis, Carnegie Institute of Technology, 1959.

[6] R. M. Cyert, et al, "Models in a Behavioral Theory of the Firm," Behavior Science (April, 1959), pp. 81-95.

to attain their profit goals.

Quite similar to this group of simulation models are the business games such as the Carnegie Tech Mark 1.5 game, in which an oligopolistic market structure is simulated and players or decision makers act within this simulated environment. Hypotheses concerning the behavior of individual players or decision systems as a whole can then be tested.[7] Gaming, however, is different from the wholly computer type simulation under discussion here. Martin Shubik states, ". . . a gaming experiment will always include human decision makers whose behavior is to be influenced by the immediate circumstances, whereas in a simulation either the behavior of a system or the behavior of individual components is taken as given . . ."[8]

The third group treats an industry as an entity. An example of models in this group is Cohen's simulation of aspects of the shoe, leather, and hide industries[9] based on

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[7]For a general discussion on business games see <u>Proceedings of the Conference on Business Games as Teaching Devices</u> (Tulane University, 1961).

[8]Martin Shubik, "Simulation of the Industry and the Firm," <u>American Economic Review</u>, Vol. D (Marck-Dec. 1960), p. 910.

[9]K. Cohen, <u>Computer Models of the Shoe, Leather, Hide Sequence</u> (Englewood Cliffs, 1960).

information provided by the study of Ruth P. Mack.[10] Cohen devised a difference equation system representing the aggregate customer expenditure on shoes and the behavior of shoe retailers, shoe manufacturers, tanners, and hide dealers. He also compared the advantages and disadvantages of using "process models" in which the values of the lagged endogenous variables, after initialization, are produced by the system, and "one-period-change models" in which the values of the endogenous variables are brought in at each successive stage of their observed values.

Another model from this group is Hoggart and Balderston's simulation of the west coast lumber industry,[11] in which the industry is divided into three levels: the supplier firms, the wholesale intermediaries, and the customer firms. There were simulated information networks, commodity shipments, cash payment flows, and the birth and death processes for firms at all levels. They also analyzed the effect of changes in the structure of the information system and the costs of information.

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[10] R. P. Mack, "A Case Study of the Shoe, Leather, Hide Sequence," Consumption and Business Fluctuations (New York, 1956).

[11] A. C. Hoggatt and F. E. Balderston, "Models for Simulation of an Intermediate Market," presented to the Economic Society (Chicago, Ill.: Dec. 29, 1958).

While models are being built to simulate the real situation, simulation languages are also being developed in order to facilitate researchers in the study of economic systems; three of these are J. W. Forrestor's "Dynamo,"[12] Markowitz's "Simscript,"[13] and Gordon's "General Purpose Systems Simulation Program."[14]

In summary, computer simulation is a new way of analyzing and interpreting models derived from empirical investigation. Once a simulation model is built, it is amendable to manipulation, merely by changing its parameters, thus allowing the system to be analyzed and modified without having its actual operations interrupted or changed. Also, because of the fast computational speed of the machine, researchers will be able to try out new ideas in compressed time, resulting possibly in substantial cost reduction.

Purpose

This thesis concerns itself with the study of the

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[12] J. W. Forrestor, Industrial Dynamics (New York: The M.I.T. Press, John Wiley, 1961).

[13] H. M. Markowitz, B. Hausner, H. W. Karr, Simscript - A Simulation Programming Language (Santa Monica, Calif.: The Rand Corporation, 1962).

[14] Geoffrey Gordon, A General Purpose Systems Simulation Program (Yorktown Heights, New York: International Business Machines Corporation, Advanced Systems Development Division, 1961).

simulation of certain stochastic relationships which can be found in most micro-economic systems. The term "stochastic"[15] implies that the values which measure the relationship may not always remain the same, but may fluctuate according to some probability distribution. Some of these relationships are described in the following.

A micro-economic system is composed of productive facilities which produce economic goods or services to satisfy human demand. The inputs to the system may be raw materials requiring processing or human beings needing services. The demand for the good or services with respect to time may be constant, may fluctuate according to some known probability distribution, or may occur randomly. Additionally, the time period required to produce a unit of a good or service depends on the availability and arrangement of the facilities in the system, and this time interval may also be constant, may fluctuate either according to some known probability distribution, or may occur randomly.

If the rate at which demand becomes effective is greater than the rate at which the supply is forthcoming,

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[15]"Stochastic comes from the Greek 'stokhos' (a target, or bull's-eye). The outcome of throwing darts is a stochastic process that is to say, fraught with occasional misses . . . Its opposite is exact or systematic." See Valvanis, <u>Econometrics</u> (New York: McGraw-Hill, 1959), p. 4.

then either some of the demand will not be satisfied or long waiting time will occur. If the rate of supply exceeds that of demand, then either there will be over-production or some of the facilities will be kept idle from time to time.

In this way, a micro-economic system may be considered as a waiting-line or queuing system, with the service facilities arranged in series, in parallel or mixed. A waiting line is formed when either units requiring service wait for service or the service facilities stand idle and wait for input units.

In this study the costs of waiting time and idle time are the points of major interest. The proposed objective is to determine that set or combination of facilities, input and production or service rate which will minimize the total cost of waiting and idle time incurred during the production of a given quantity of goods or services. Micro-economic systems with both single-channel and multichannel service or production stations will be simulated and sample runs will be performed on an electronic digital computer with the intention of inferring the simulation results to those of a real situation. The following is the general procedure used in this study.

Procedure

(1) Formulation of the problem

The problem to be explored and the questions to be answered must be determined and clearly defined at the start. In micro-economics, the basic problem is the optimum allocation of scarce and finite resources among competing enterpreneurial goals. In the present study, this problem will be investigated by simulating a micro-economic system. The following questions are some that must be answered preparatory to formulating a model depicting this system: How is the system presently organized? What are the flows of traffic in the system? Are there any bottlenecks or areas of traffic congestion in the system? If bottlenecks or areas of traffic congestion exist, where are they and what is their effect on costs and profit? Are there any idle facilities in the system? If so, where, how many, and what are their costs? Where are the information feedback cycles and what are the decision-making processes at each level of the system?

(2) Construction of the model

Before a model, which is to represent certain aspects of a micro-economic system, can be constructed, relevant information regarding the system must be gathered. Possible kinds and sources of information include the following:

1. Verbal descriptions of the activities of each elementary part of the system and the arrangement of these elementary parts inside the system are obtainable from either the management or operating personnel.
2. The values of the parameters in the simulation model, such as the means and the variances of the assumed probability distributions of the variables, may be estimated from information obtained by sampling different system processes.
3. Analytical solutions of the behavior of some elementary parts of the system, which have already been theoretically or empirically verified and proved, can be adopted and integrated into the main simulation program.

When enough information relevant to the purpose of the study has been collected and analyzed, a block diagram embodying the logic and action of the model will be produced. This diagram will indicate the physical flows of the input units and the flows of information in the system. When a block diagram is drawn, the model is considered as having been built.

It should be made clear that a simulation model of a micro-economic system is not an exact replica of the real environment. Only certain aspects of the system relevant to the problem under consideration are simulated. The reason, besides the financial one, is to keep the researcher from being overly burdened with facts; otherwise, the points which need to be illustrated might become obscured.

John Harling,[16] in a paper reviewing simulation techniques in the area of operations research, made the following comment: ". . . The construction of a model of the real system should neither oversimplify the system to the point where the model becomes trivial, nor carry over so many features from the real system that the model becomes intractable and prohibitively clumsy . . ."

(3) Formation of a computer program

After a block diagram is drawn, the model is then described through a well-defined computer language, thus becoming a computer program. In this study, the IBM 1410 FORTRAN will be used as the language for this purpose.[17] The activities of each elementary part of the model will be described by an individual subprogram which becomes an

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[16]John Harling, "Simulation Technique in Operations Research - A Review," Operations Research, Vol. VI (May-June, 1958), p. 307.

[17]Neither SIMSCRIPT nor GPSS languages are available for the I.B.M. 1410 system.

entity and which can be modified as the situation warrants. These subprograms can be considered as the building blocks of the main program, the whole of which describes the model representing the system. While integrating the subprograms into one main program, emphasis will be placed on queues, or waiting lines, the arrangement of service stations or facilities, information feedback loops, and decision-making processes which affect and are affected by environment.

The information on which the simulation is based may change quite often; the input rate may change due to the change of the demand and the service rate may change due to the improvement of technology. This problem, however, can be easily handled, since the number of subprograms can be altered and parameters modified prior to any computer run. Thus, the building block method of program construction provides adequate flexibility.

(4) Evaluation of the system

After examining the results obtained from simulation runs[18] with the computer program which describes the

[18]"Simulation run" is defined by Guy H. Orcutt as "An individual simulation run may be thought of as an experiment performed upon a model . . . Given completely specified initial conditions, parameters and exogenous variables, a single simulation run yields a single set of time paths of the endogenous variables." See G. H. Orcutt,

model, some of the following interesting facts will be shown. First, the area or areas where the flow of traffic is delayed or where capacity becomes idle can be determined. Second, if the unit cost of idle time and waiting time can be ascertained, the profit significance of the system's behavior can be assessed. A more detailed discussion of this subject is found in Chapter V. Third, experimentation with the model will expose the interrelationship between parts of the system and thus may indicate alternative ways of organization and may, additionally, generate new ideas to improve the system. And fourth, by evaluating the relative risks of both bold and conservative courses of action, the user of the program may gain a greater appreciation of the risk involved in each alternative decision.

There is one word of warning, however, Periodic re-examination of the parameter values and the basic assumptions involved in the simulation model is very important in making the program always useful and current.

"Simulation of Economic Systems," The American Economic Review, Vol. 50 (Dec. 1960), p. 893. And since the simulation run is on a computer, the term "computer run" is used analogously as "simulation run" in this thesis.

(5) Suggestings regarding the improvement of an existing system or the design of a new system

After system evaluation, the user of the program can then proceed to find ways to bring into optimum balance the many diverse objectives involved. For example, the objective may be to minimize cost; then if the relationship of cost to time were known, several adjustments might be made to improve the efficiency of an existing system. For example, the simulation runs will indicate the areas where facilities are often lying idle and where the flow of traffic is often interrupted because of a shortage of facilities. By internal re-arrangement of these facilities, which is the movement of facilities from slack areas to congestion areas or the change of some series arrangement of facilities to parallel arrangement, the waiting time and the idle time may be reduced, thereby reducing the total cost of waiting time and idle time.

In addition, the simulation runs will show whether the congestion areas are causing other parts of the system to become idle, thus prolonging the total service time and causing fluctuation in the rate of the traffic flow, and thereby increasing total cost. In some cases, it may be the cause for losing a part of the effective demand. Therefore, evaluation should be made to see whether it would be more profitable to hire additional resources from outside of the system and apply them to the more sensitive areas in the system.

A further example of model evaluation can be seen in the following: If internal rearrangement of facilities and hiring additional resources from outside are impractical due to other considerations, the results of simulation runs will at least indicate where effort should be directed to finding ways of speeding up the service rate, information relays, and decision-making processes.

In designing a new system, it would be advisable to build a simulation model first. The simulation runs will show the behavior and characteristics of the new design which otherwise may not be foreseen until the system is actually organized and in operation. Having made repeated simulation runs and modifications of the design, by the time the real system is organized and constructed, many of the flaws of the original design may already have been corrected. Thus, the simulation model will provide the user with a working analogy to test the implication of new plans, new schemes, and new ideas. And because the simulation runs will reveal the interrelationships between all the elementary parts and the subsystems in a micro-economic system, management control can be designed and established to check the actual output at each step against the desired output; and control devices can be installed to hold the operational variables within desired limits.

The General Motors Corporation has built a simulation model of the inventory control procedures to test new ideas.[19] The effort is properly rewarded, because the experimentation of the model gives the management many theoretical insights as to the sensitivity of the system to variables, which can be controlled by suitable management decisions.

Outline

In this chapter, the concept of computer simulation and its application in economics has been reviewed and discussed. In addition, a general procedure of computer simulation of economic systems has been presented.

In the following chapter, a method of computer simulation of the stochastic relationships in micro-economic systems is introduced. In Chapter III, two simple waiting line models are constructed using this method and the results obtained from the simulation runs are compared to the

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[19] E. Naddor, "Markov Chains and Simulations in an Inventory System," The Journal of Industrial Engineering (March-April, 1963), Vol. XIV, Number 2, p. 91.

analytical solutions to insure the validity of the method.

In Chapter IV, the same method is used in the simulation of more complicated waiting line models for which analytical solutions have not yet been found. Chapter V attempts to draw general conclusions about the behavior of waiting line systems from the results of the simulation runs, and the economic significance of the conclusions is discussed.

In the concluding chapter, extension of the method to the simulation of other aspects of micro-economic systems is explored and illustrated.

All the block diagrams and IBM 1410 FORTRAN programs of the models are to be found in the appendixes.

CHAPTER II

MODEL CONSTRUCTION

The purpose of this Chapter is to set forth the procedures for constructing a simulation model of a microeconomic system. The presentation will consist of a detailed discussion of the use of subprograms to simulate the behavior of the elementary parts of the system and a main program to simulate the behavior of the whole system. Using information obtained from an existing system or assumptions made regarding a hypothetical or a design of a system, a block or flow diagram is prepared which will indicate the structure of the system and the interrelationships among its parts.

If the quantities which measure the behavior of the elementary parts of the system, such as the interarrival time intervals and the service time intervals at each service point in the system, or weights or volumes of the input raw materials and output products, are stochastic in nature, computer programs can be written to simulate these stochastic relationships. A main program, utilizing the subprograms at appropriate places, can then be written to simulate the whole system.

Once the main program is completed, experimentation with the model is accomplished by varying the parameters of the system, typical parameters being the means and variances of input and service rates, or the means and variances of the conversion rates of raw materials to finished products. The computer will then, for each set of parameter values, go through the step-by-step procedure specified by the simulation programs.

Subprograms

Subprograms are written to simulate the activities in each elementary part of the micro-economic system. In this investigation, the time interval between arrivals and the time interval that each unit stays in each elementary part is the point of interest. The technique of simulating these time intervals is as follows.

Sample observations of the time intervals (t) actually occurring in the real system are made and then organized in a frequency distribution. A theoretical probability distribution such as a normal, exponential, or Gamma distribution is selected and compared with the empirical distribution. Goodness of fit testing[1] may be used to determine whether any of the theoretical distributions can be accepted as representing the population distribution. If none of the theoretical distributions can be fitted to the empirical distribution, least-squares or other estimation methods may be used to find the curve which best approximates the frequency distribution of the sample observations. The procedure of using the least-squares method to find the curve is sometimes referred to as ". . . the polynomial approximation to the distribution."[2] The curve thus obtained may be considered as an approximate representation of the population distribution. Having then a suitable representative distribution, the Monte Carlo method of sampling[3] may be used to generate the appropriate random variate.

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[1]See any statistical textbook of intermediate level, such as Croxton and Cowden, Applied General Statistics (Englewood Cliffs, New Jersey: Prentice-Hall, 1958), Chapter 25.

[2]John Harling, "Simulation Techniques in Operations Research - A Review," Operations Research, Vol. VI (May-June, 1958), p. 307.

[3]The range of the cumulative probability function for any probability distribution is from 0 to 1. The Monte Carlo method of sampling is to generate a random number in the range of 0 to 1, representing the cumulative probability function, and the corresponding value of the variable is a sample value. See Sasieni, Yaspan, Friedman, Operations Research - Methods and Problems (New York: Wiley), p. 58.

Generation of Random Numbers

In order to generate a random variate, it is first necessary to generate a sequence of random numbers. Of course, the easiest way of accomplishing this is to utilize a random number table. However, when a computer is being used, storage restrictions often rule out the possibility of maintaining an adequately sized table. The alternative is to generate the random numbers in the computer. This latter process has one obvious advantage; namely, the properties of the sequence of the random numbers generated are known prior to using the numbers in a particular problem. The method used in this paper is adopted from the IBM reference manual for random number generation and testing.[4] The procedure is as follows.[5]

Let d be the number of digits contained in the random number variate. N, the starting value of a random number sequence, be an integer not divisible by 2 or 5, and M be the constant multiplier which is equal to 200 K $\pm$ r, where K is an integer, and r is any of the following values: 3, 11, 13, 19, 21, 27, 29, 37, 53, 59, 61, 67, 69, 77, 83, 91 (a value of M close to $10^{d/2}$ is a good choice).

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[4] IBM Reference Manual, "Random Number Generation and Testing," 1959.

[5] The block diagram of the procedure is shown in figure (1) of the Appendix.

The product of N and M is a 2 d-digits number. The higher order d-digits are discarded and the lower order d-digits are the value of the next random number. Each successive random number is then obtained from the lower order d-digits of the product ($N_i \times M$), N_i being the preceding random number. The procedure will produce $5 \times 10^{d-2}$ pseudo-random numbers before repeating for d greater than 3.[6]

Generation of Random Variates[7]

Since the range of the cumulative probability function of any probability distribution of a variable is from 0 to 1, random numbers ranging over this interval may be generated representing a set of cumulative probabilities. The values of the variable corresponding to the cumulative probabilities therefore may be considered as the random variate taken from a particular distribution. The cumulative probability function may be stored in the memory of a computer and the random number may be used as the argument in a "table look-up" routine[8] to find the corresponding

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[6]This process is known as the generation of pseudo-random digits by determinate sequences.

[7]Each occurrence of the variable is called a variate.

[8]A table look-up routine is to compare a single number with a list of numbers arranged in either ascending or decending order; when the number of the list equals to this single number, the comparison stops and the respective position of this single number on the list is found.

random variate. However, if the memory space of the computer imposes a limitation, then the Monte Carlo method of sampling is usually preferred.

The method for generating random variates for a given distribution depends on the particular feature of the distribution and the way in which it is related to other distributions. The following are methods for generating the random variates of some of the theoretical and non-theoretical distributions used in this study.

(1) Generation of a uniformly distributed Variate[9]

The density function of a uniform distribution is $f(x) = \frac{1}{B-A}$, where A is the lower limit and B is the upper limit of the variable x. (2a)

The mean and variance of x are $\frac{B-A}{2}$ and $\frac{(B-A)^2}{12}$ respectively. The distribution implies that x can be any of the values included in the interval specified by A and B with equal probability.

Given A and B as the lower and upper limit, and R a random number ranging from 0 to 1, the uniformly distributed variate x may be generated by the following formula.

$$x = A + (B - A) \cdot R \; ; \; 0 \leq R \leq 1 \qquad (2b)$$

[9] The procedure for generating this variable is shown by block diagram in figure (2) of the Appendix.

(2) Generation of Exponentially distributed Variate[10]

The density function of an exponential distribution is $f(x) = Ae^{-Ax}$, where A is a positive constant and the range of the variable x is from 0 to infinity. (2c)

The mean of x is $\frac{1}{A}$ and its variance is $\frac{1}{A^2}$. The cumulative probability function of x is $P(x) = 1-e^{-Ax}$. Since the range of the cumulative probability function is from 0 to 1, the value of $(1 - e^{-Ax})$ will increase from 0 to 1 as x starts at zero and goes to infinity, or the value of e^{-Ax} will decrease from 1 to 0, as x starts at 0 and goes to infinity.

Consider the following: Let $R = e^{-Ax}$, where R is a random number in the unit interval, then $\log R = \log e^{-Ax} = -Ax$. Thus, the variate x of the exponential distribution, with mean and standard deviation both equal to T, can, therefore, be generated by the following formula:

$$x = -\frac{1}{A} \log R = -T \log R. \qquad (2d)$$

(3) Generation of the Variate of a kth Erlang Gamma Distribution[11]

The density function of an Erlang Gamma distribution is $f(x) = \frac{A^k}{(k-1)!} x^{k-1} e^{-Ax}$, where A is a constant and k is

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[10]The procedure for generating this variate is shown by block diagram in figure (3) of the Appendix.

[11]The procedure for generating this variate is shown by block diagram in figure (4) of the Appendix.

an integer. The range of x is from zero to infinity. (2e) If there are k random variables, namely t_1, t_2, . . t_k, which are independent and with a common exponential distribution with mean equal to $\frac{1}{A}$ and variance equal to $\frac{1}{A^2}$, then the random variable $(t_1 + t_2 + . . t_k)$ follows the kth Erlang distribution with mean equal to $\frac{k}{A}$ and variance equal to $\frac{k}{A^2}$.

Thus, the variate of a kth Erlang Gamma distribution may be generated by the following formula:

$$x = \sum_{i=1}^{k} t_i = - \sum_{i=1}^{k} \frac{1}{A} \log R_i \qquad (2f)$$

where the R_i are random numbers ranging from 0 to 1.

(4) Generation of normally distributed Variate[12]

The density function of a normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (2g)$$

where μ and σ^2 are, respectively, the mean and variance. The range of the variable x is from $-\infty$ to $+\infty$.

A normally distributed variate may be generated by summing short sequences of uniformly distributed variables.[13]

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[12]A normal distribution is a symmetric distribution which usually has smaller variance than an exponential distribution of the same mean. The procedure of generating a normally distributed variate is shown by block diagram in figure (5) of the Appendix.

[13]The central limit theorem states that the distribution of the sample means is a normal distribution.

For the sake of simplifying the calculation, a uniform distribution with its variable y ranging from 0 to 12 is chosen. The mean μ_y and variance σ_y^2 are 6 and 12. Samples of 12 observations (N) each are taken from the distribution. The standardized normal variate of the distribution of the sample means, therefore, equals the following:

$$z = \frac{\bar{y}-\mu_{\bar{y}}}{\sigma_{\bar{y}}} = \sum_{i=1}^{12} R_i - 6 \ ; \ 0 \leqq R_i \leqq 1 \qquad (2h)$$

Let μ and σ be the mean and standard deviation of a normal distribution, then the variate of the distribution may be generated by the following formula.

$$x = \mu + \sigma \cdot z \qquad (2i)$$

where z is the standardized normal variate generated by the preceding formula.

(5) Generation of a lognormally distributed Variate[14]

The density function of a lognormal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2}$$

$$\text{where } \mu = \frac{\sum_{i=1}^{N} \log x_i}{N}, \quad \sigma^2 = \frac{\sum_{i=1}^{N} (\log x_i - \mu)^2}{N},$$

$$\text{and } o \leqq x \leqq \infty \qquad (2j)$$

A lognormal distribution will become a normal distribution, if the abscissa scale is logarithmic. Therefore, the

[14]The procedure for generating this variate is shown by block diagram in figure (6) of the Appendix.

relationship between the mean μ and variance σ^2 of a log-normal distribution and the mean μ_y and variance σ_y^2 of the normal distribution can be expressed by the following:

$$\mu_y = \log \mu - \frac{1}{2} \log \left(\frac{\sigma^2}{\mu^2} + 1\right)$$

$$\sigma_y^2 = \log \left(\frac{\sigma^2}{\mu^2} + 1\right) \qquad (2k)$$

Thus, a lognormally distributed variate x may be generated by the following formula:

$$x = e^{\mu_y + \sigma_y \cdot Z} \qquad (21)$$

where z is the standard normal variate.

(6) Generation of variate of a sinusoidal function[15]

If the mean of a probability distribution does not remain constant during the passage of time but instead fluctuates according to some trignometric function, with constant period, then the mean μ of this function can be determined at any particular point of time T by the following formula:

$$\mu = \mu_0 + A \cdot (\mathrm{Sin}\, \theta) \;, \; \theta = \frac{T}{P} \cdot 2\pi \qquad (2m)$$

Here, μ_0 is the initial value of the mean, A the amplitude, and P the period of the cycle. θ therefore will take on

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[15] The procedure for generating this variate is shown by block diagram in figure (7) of the Appendix.

values ranging from 0 to 2π.

A more complex situation is one in which the value of the mean oscillates according to more than one trignometric function, each with different periods and amplitudes. One such situation can be represented by:

$$\mu = \mu_0 + (A_1 \text{ Sin } (\frac{\theta}{w_1}) + A_2 \text{ Cos } (\frac{\theta}{w_2})) \qquad (2n)$$

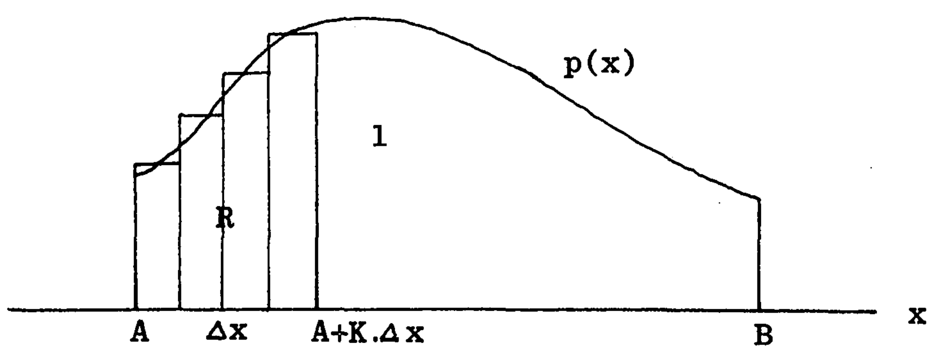
where w_1, w_2 are the different angular velocities.

(7) Generation of the variate of a non-theoretical distribution

If none of the known theoretical distributions can be accepted as a close approximation of the empirical distribution obtained from random sample observations, then numerical integration may be used to generate the random variate of the distribution.

Figure 1.

Diagram of the Numerical Integration of a Probability Function



Consider the above diagram, where A is the lower and B the upper limit of the range of the variable x. Call p(x) the probability distribution of x obtained by polynomial approximations of empirical observations. Then the following is known:

$$\int_{-\infty}^{A} p(x)\,dx = 0 \ , \quad \int_{A}^{B} p(x)\,dx = 1 \ , \quad \int_{B}^{+\infty} p(x)\,dx = 0 \qquad (20)$$

To generate a sample value of x, a random number R with values ranging from 0 to 1 is first generated. Then a numerical integration method, such as the trapezoidal rule,[16] may be used to integrate the area under the curve p(x) starting from A. When it reaches a point where the area under the curve is equal to the random number, the corresponding value on the abscissa is a sample value of the variate.

When sample values of a distribution have been generated by using the methods described above, their acceptability may be examined by looking at the moments of the sample distribution.

The theory underlying the testing whether a sample of size N can be accepted as being taken from a given

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[16] See any text book on numerical methods, such as M. G. Salvadori and M. L. Baron, Numerical Methods in Engineering (New Jersey: Prentice-Hall, Inc., 1961), p. 89.

distribution is that if the sample is randomly drawn from a known distribution with mean equal to $\bar{\bar{X}}'$ and standard deviation equal to σ', then the sample mean $\bar{\bar{X}}$ will be included in the interval $\bar{\bar{X}}' \pm Z \frac{\sigma'}{\sqrt{N}}$ where $\frac{\sigma'}{\sqrt{N}}$ is the standard error of the mean, with a probability set by the value of Z. For example, to establish a 95 per cent confidence interval, Z is set to be 1.96. Also, when the sample size exceeds 30, the sample standard deviation σ can be assumed to be normally distributed about σ' with standard error equal to $\frac{\sigma'}{\sqrt{2N}}$. Thus, σ will be included in the interval $\sigma' \pm Z \frac{\sigma'}{\sqrt{2N}}$ with a probability set by the value of Z. Once the standard errors are determined, the required sample size can be calculated. If it is decided that the sample mean $\bar{\bar{X}}$ should fall inside the interfal $\bar{\bar{X}}' \pm A$[17] with a probability of .95, then the required sample size N should be set approximately equal to $(\frac{Z\sigma'}{A})^2$, and if it is decided that the sample standard deviation should fall inside the interval $\sigma' \pm B$ with a probability of .95, then the required sample size N should be set approximately equal to $(\frac{Z\sigma'}{\sqrt{2}B})^2$ where Z is set to be 1.96.[18]

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[17]Sometime A is expressed as a percentage of $\bar{\bar{X}}'$, for example, $A = 0.1\bar{\bar{X}}'$.

[18]See any textbook on statistics and quality control, such as A. J. Duncan, Quality Control and Industrial Statistics (Homewood, Illinois: Richard D. Irwin, Inc., 1959), p. 49.

Main Program

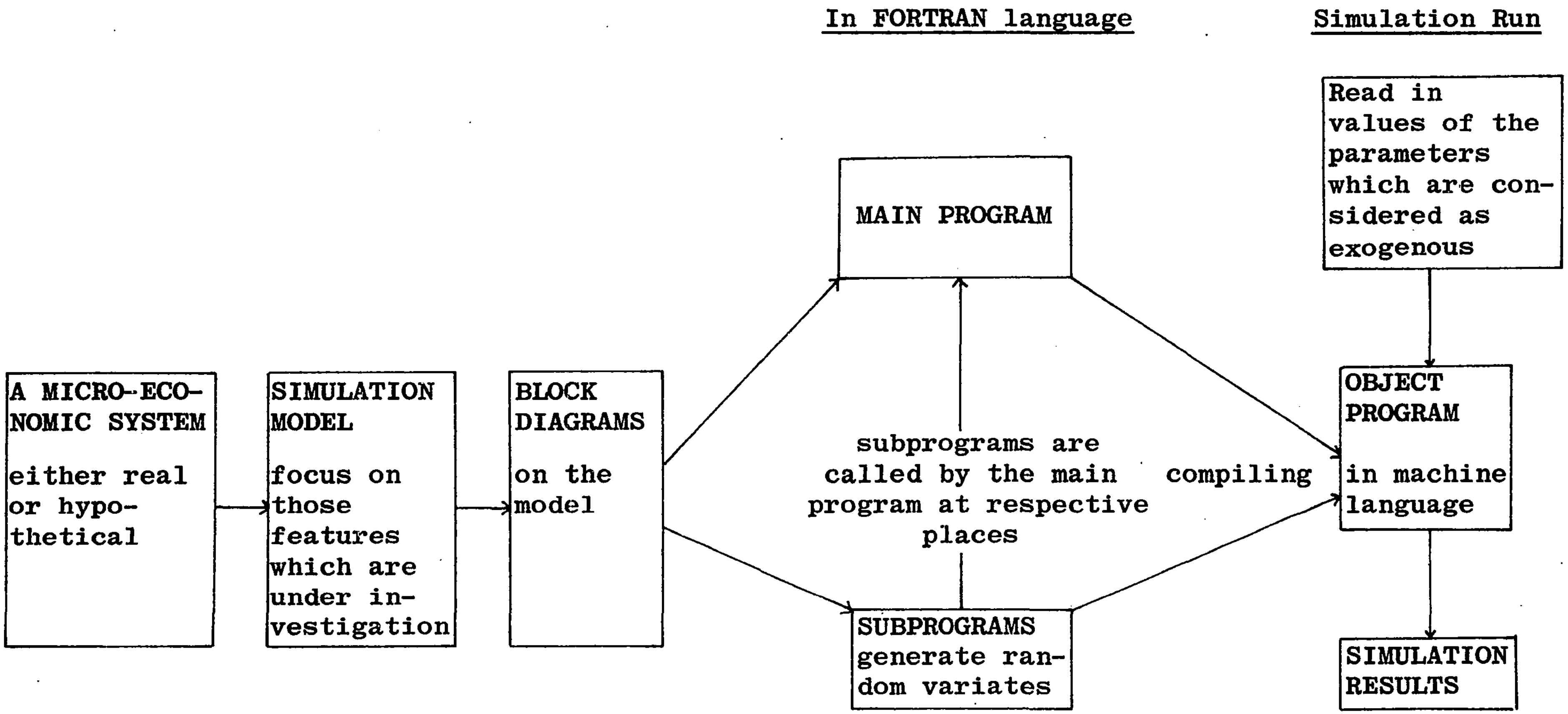
The main program is written to simulate the structure and activities of the entire system. This structure depicts how the various elementary parts are arranged and connected to each other. Additionally, the main program will contain "call" statements which will call the respective subprograms at those places where the behavior or activities of the elementary parts are to be simulated. Thus, subprograms are the building blocks of the main program.

The activities of the micro-economic system simulated in this study are the time dimensional flows of the input units through the entire system, whether waiting to be admitted or being served in the elementary parts of the system. A flow starts when a unit arrives at the system and ends when the unit leaves the system. The congestion or discontinuity of the flows and the rates of the flows or the length of the time intervals that the input units spend in each elementary part of the system will be recorded. In addition, the waiting time of the input units and the idle time of each elementary part will be calculated and analyzed.

The entire procedure of the computer simulation adopted in this paper is reviewed and illustrated in figure (2).

Figure 2.

Arrow Diagram of the Procedure of Computer Simulation



Two different methods of computer simulation have been tried in this investigation to simulate the time dimensional flows inside the micro-economic system. The remainder of this chapter will consider these methods.

Methods of simulation

The first method can best be described by the following block diagram, Figure (3).

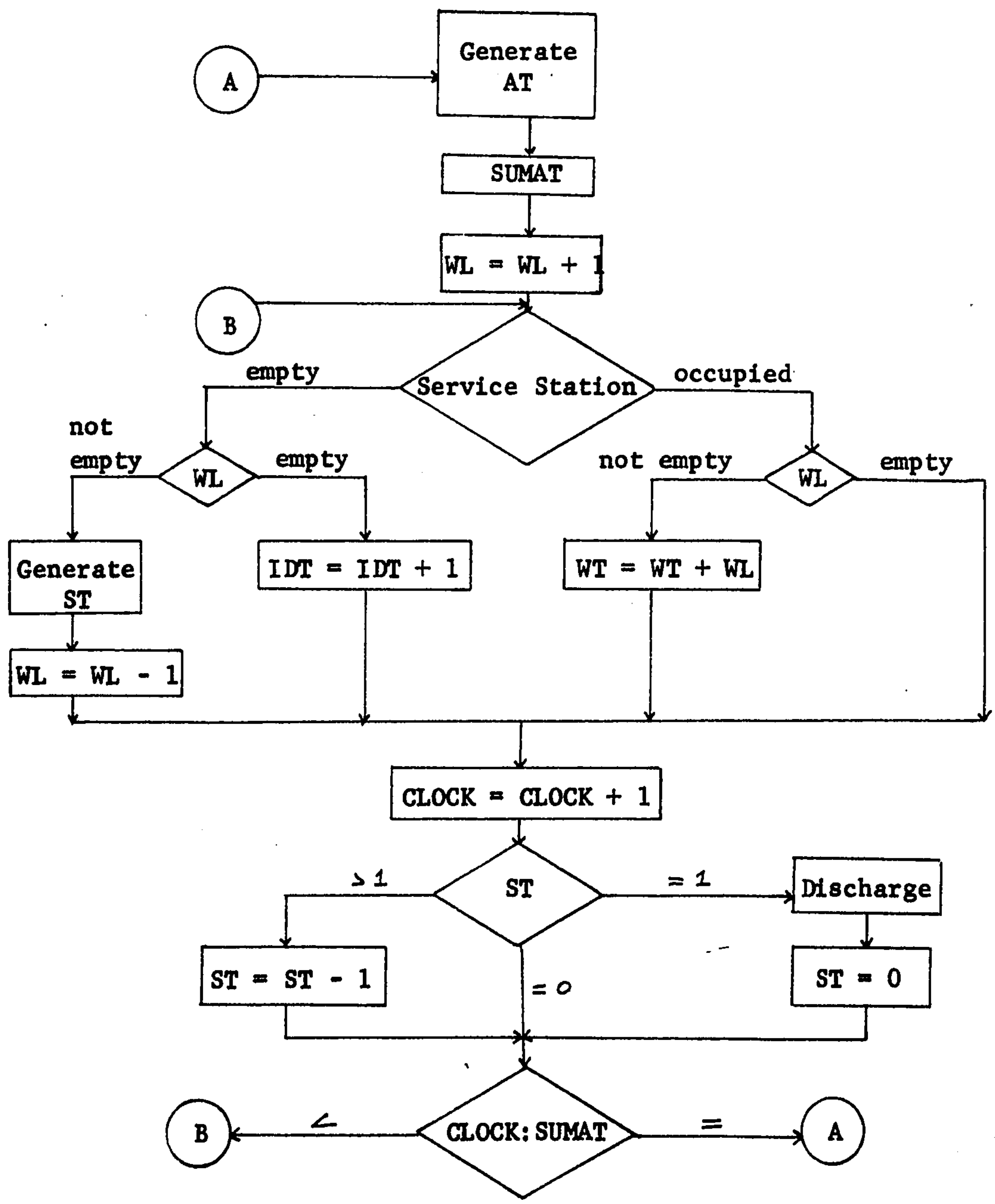
The diagram shows that an interarrival time interval is first generated[19] and the arrived unit is put in the waiting line. Next, a check is made to determine whether the service station is occupied or empty at the moment. If the service station is empty, a test is made to ascertain if a unit is now waiting to enter the service station; if there is none, a unit of waiting time is recorded. If a unit is waiting, it enters into the service station and a service time interval is generated, while simultaneously, the waiting line is reduced by one. On the other hand, if the service station is occupied, a test is also made to determine whether there is any unit waiting. If no unit is waiting, no waiting time is recorded; if there are units waiting, each unit in the waiting line causes the total waiting time to be increased by

[19] Time intervals are generated by the computer subprograms presented earlier in the chapter.

Figure 3.

Block Diagram of Simulation Method 1

AT = Interarrival time interval
ST = Service time interval
WL = Number in waiting line
WT = Waiting time
IDT = Idle time



one time unit.

The next step is to advance the clock by one time unit and check the service time interval. If the interval equals one, then the service on that particular unit is completed, a discharge is therefore recorded, and the service interval is reduced to zero. If the interval is equal to zero, indicating that the station is empty, no action is taken. If the interval is greater than one, it is decreased by one time unit.

The next step is to compare the clock time with the interarrival time. If they are equal, indicating that the next unit has just arrived at the service station, another interarrival time interval is generated and the new arrived unit is put in the waiting line. If the clock time is shorter than the next arrival time, the process is to go back to check the service station again. This process is carried on until the last input unit has arrived and been served.[20]

The second method is described by the following block diagram, Figure (4).

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[20]This method is called the fixed time increment method (used in the General Purpose Systems Simulation Program) and presupposes discrete or rounded service and interarrival time distributions.

Figure 4.

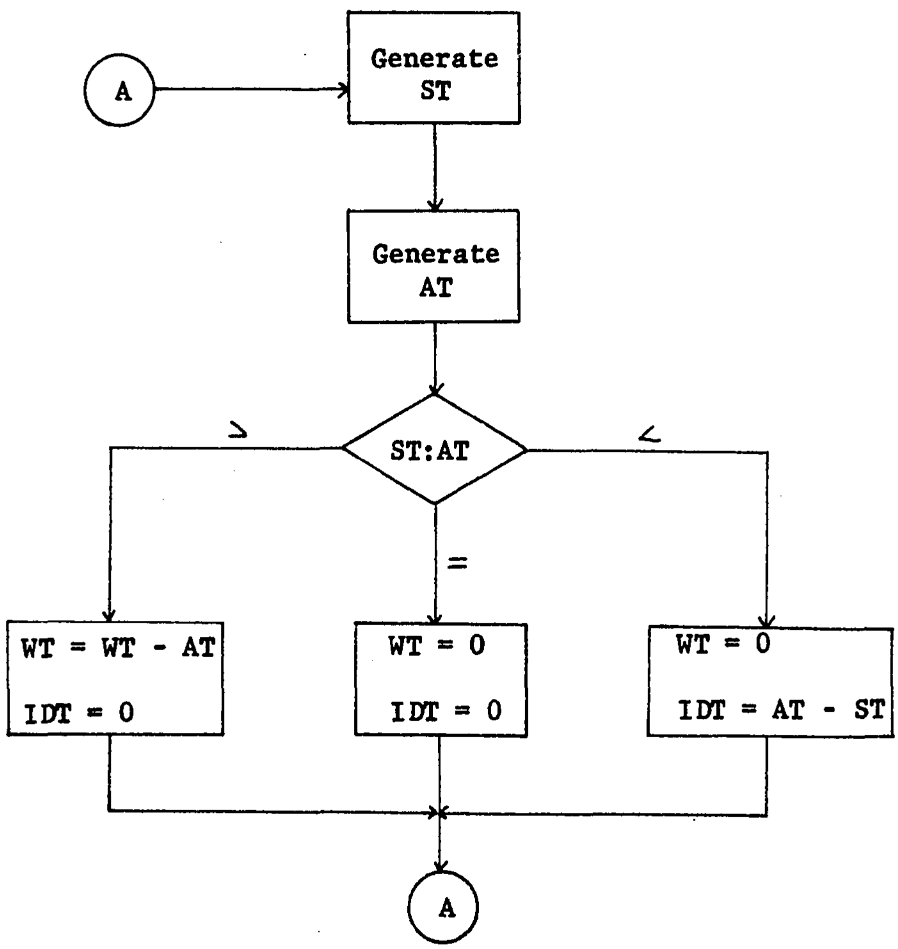
Block Diagram of Simulation Method 2.

AT = Interarrival time interval

ST = Service time interval

WT = Waiting time

IDT = Idle time



According to the diagram, immediately after a service time interval has been generated, an interarrival time interval is generated to indicate the arrival time of the next input unit. After comparing the service time interval with the interarrival time interval, if the service time interval is longer, then waiting time has occurred and the absolute difference of the two intervals is the length of the waiting time. If the interarrival time interval is longer, then idle time has occurred and the absolute difference of the two intervals is the length of the idle time. If the two intervals are equal, neither waiting time nor idle time has occurred.

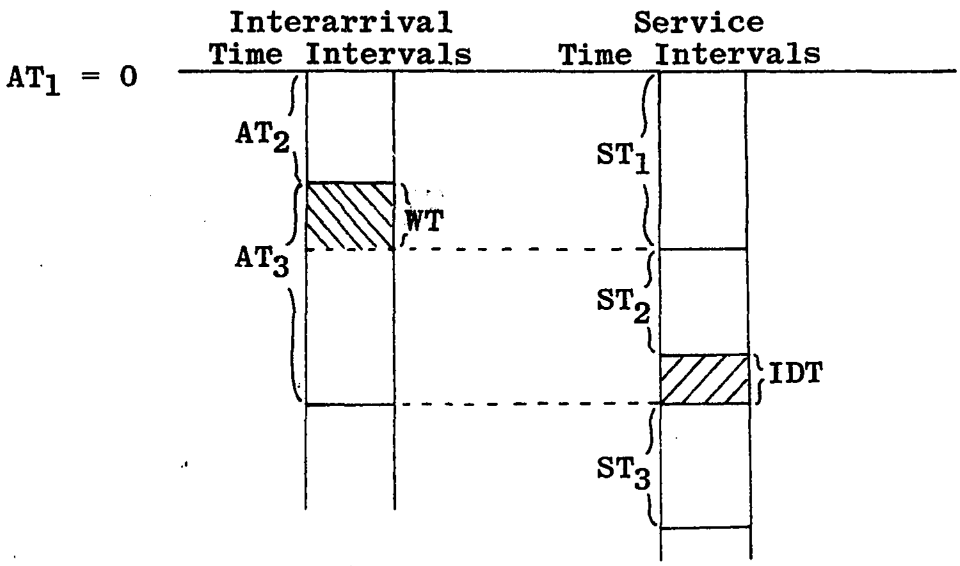
The next step is to go back to generate another set of service time interval and interarrival time interval. This process is carried on until all the input units have arrived at the service station and been served.

All the information about the behavior of the flows is stored in the memory of the computer and is retrievable at the end of each simulation run.

Figure (5) gives another illustration of the method. Using the same abbreviation, AT1 is the interarrival time interval between input unit 1 and 2, and ST1 is the service time required for unit 1. Since ST1 is longer than AT1, waiting time occurs and WT2 indicates the length of the

Figure 5.

Subsidiary Diagram of Simulation Method 2



waiting time. AT2 is the interarrival interval between unit 2 and 3 and ST2 is the service time required for unit 2. As ST2 is shorter than AT2, idle time occurs and IDT is the length of the idle time.

After some preliminary trials, it was found, however, that when the first method was used in constructing a model, the time necessary for a simulation run was much longer than that for the second method, because the "clock" advances only one discrete time unit at each step. Therefore, the second method, which may be called the variable time increment method, will be used in the construction of all the simulation models presented in this thesis.

Summary

In this chapter, a method of constructing the simulation model of elements of a micro-economic system has been presented, with major interest placed on the simulation of the behavior of the flows of the input units passing through the system. These flows are simulated through the use of a computer main program and several computer subprograms. The main program describes the structures of the system and the relationship between its elementary parts, and the subprograms generate the respective random variates of the probability distributions being used.

In the following chapter, the micro-economic system is represented by two simple waiting-line models, which are constructed according to the method presented in this chapter and sample simulation runs are made to test the simulation results with analytical solutions obtained by solving the mathematical equations representing the same models in order to assure the accuracy of the simulation method.

CHAPTER III

SIMULATION RESULTS AND ANALYTICAL SOLUTIONS

The purpose of this chapter is to test the accuracy of the simulation method presented in the previous chapter. This will be accomplished by selecting two basic waiting-line models for which, in addition to the results obtained from the simulation runs, analytical solutions can also be found. By comparing the simulation results with the analytical solutions, it can be determined whether any statistically significant difference exists between the two.

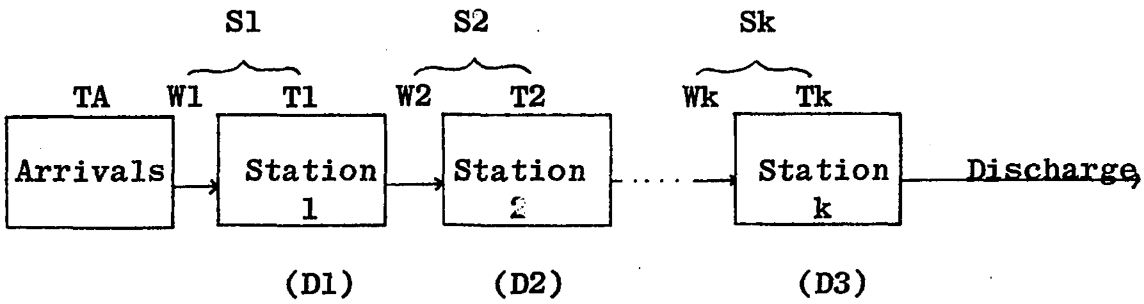
Model 1

The first model is a single-channel model which depicts a situation in which all the service stations in a micro-economic system are arranged in series. The input units pass through these service stations, one by one, in a given order. It is described by the following diagram and equations.

In the diagram, TA are the interarrival time intervals of the input units, Wi (i = 1, 2, ... k) are the waiting time intervals, Ti (i = 1, 2, ... k) are the service

Figure 6.

Arrow Diagram of A Single-Channel Model



time intervals, Di $(i = 1, 2, \ldots k)$ are the idle time intervals, and Si $(i = 1, 2, \ldots k)$ are the sums of Wi and Ti. k is the total number of service stations in the system.

For the first arrival of the input units, the following equations are applicable:

$$(TA)_1 = 0;$$

$$(Wi)_1 = 0,\ (W2)_1 = 0,\ .\ .\ .,\ (Wk)_1 = 0;$$

$$(Di)_1 = 0,\ (D2)_1 = 0,\ .\ .\ .,\ (Dk)_1 = 0;$$

$$(S1)_1 = (T1)_1,\ (S2)_1 = (T2)_1,\ .\ .\ .,$$

$$(Sk)_1 = (Tk)_1 \qquad (3a)$$

For subsequent arrivals, the equations are changed to the following, where $n = 2, 3, \ldots, N$, N being the total number of the input units.

$$(S1)_n = (W1 + T1)_n$$

$$(S2)_n = (W2 + T2)_n$$

.
.
.

$$(Sk)_n = (Wk + Tk)_n \tag{3b}$$

Whether waiting time or idle time will occur depends on the differences between $(S1)_{n-1}$, $(S1 + S2)_{n-1}$, . . . $(S1 + S2 \ldots +Sk)_{n-1}$ and $(TA)_n$, $(TA + W1 + T1)_n$, . . . $(TA + W1 + T1 \ldots, +Wk + Tk)_n$. If the differences are positive, idle time will be zero, and waiting time can be calculated by the following equations.

$$(W1)_n = (S1)_{n-1} - (TA)_n$$

$$(W2)_n = (S1 + S2)_{n-1} - (TA + W1 + T1)_n$$

.
.
.

$$(Wk)_n = (S1 + S2 \ldots, +Sk)_{n-1} - (TA + W1 + T1 \ldots, +Wk + Tk)_n \tag{3c}$$

If the differences are negative, waiting time will be zero, and idle time can be calculated by the following equations.

$$(D1)_n = (TA)_n - (S1)_{n-1}$$

$$(D2)_n = (TA + W1 + T1)_n - (S1 + S2)_{n-1}$$

.
.
.

$$(DK)_n = (TA + W1 + T1 \ldots, + Wk + Tk)_n - (S1 + S2 \ldots, +Sk)_{n-1} \tag{3d}$$

If these differences are zero, both waiting time and idle time will be zero.

In waiting line problems, if the input units arrive at a service station in a random and independent fashion, they may be described by a poisson distribution with parameter λt, where λ is the average arrival rate and t is a time interval of fixed length. In other words, the distribution indicates the variation of the number of arrivals occurring in time intervals of specified length. However, if the time intervals between each two consecutive arrivals, rather than the number of arrivals within an interval, are under investigation, the appropriate distribution is a negative exponential distribution with mean equal to $\frac{1}{\lambda}$ and variance equal to $\frac{1}{\lambda^2}$. In contrast, then, this distribution describes the variation in length of the interarrival intervals. This latter distribution is also used to describe the variation in service time required to complete a service provided that the service time is a random and independently distributed variable.

In order to derive an analytical solution to the model, it is assumed that there is only one service station; in other words, k equals 1.[1] It is further assumed

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[1]The block diagram of this model is illustrated in figure (8) of the Appendix.

that (1) input units arrive at the service station in a poisson fashion with mean arrival rate, λ, equal to 1/6, and (2) the service times follow an exponential distribution, with mean service rate, μ, equal to 1/5. Since the mean of the service rate μ is greater than the mean of the arrival rate λ, the expected waiting time for the input units can be found analytically by the following well-known formula, where $\bar{E}(w)$ is the expected value of waiting time for the model.[2]

$$E(w) = \frac{\lambda}{\mu(\mu-\lambda)} = 25 \text{ time units} \tag{3e}$$

Four thousand, nine hundred[3] input units were simulated to pass through the system on the computer run of this single-channel model. The simulation results showed that the mean and standard deviation of the waiting time for these input units were 25.09 and 29.82 respectively.

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[2] Sasieni, Yaspan, Friedman, Operations Research - Methods and Problems (New York: Wiley, 1960), p. 133.

[3] Trial runs show that for generating variates of the following probability distributions, a sample of 100 input units already give high precision. The results are listed below.

	True Mean	True S.D.	Sample Mean (N=100)	Sample S.D. (N=100)	Z
Exponential	20	20	20.37	19.66	.19
Normal	20	5	19.92	5.49	-.14
Lognormal	20	5	20.01	5.18	.02

Here, a large sample of 4,900 input units was used in order to assure greater precision.

The standard error therefore is .42.[4] The difference between the simulation result and the analytical solution is .09. The Null hypothesis that this difference is statistically significant was rejected at the .05 level. Thus, it is interpreted that the difference is due to random perturbation.

Model 2

The second model is a multi-channel model which depicts a situation in which the service stations in a microeconomic system are arranged in parallel,[5] such as in a barber shop.

Here, each input unit may pass through any one of the service stations, provided that the service station is empty at the moment. In the simulation program, the number of channels is a parameter which is set by the user at the beginning of each simulation run. A graphical representation of the simulation procedure is illustrated below, using a three-channel model as an example.[6]

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[4]Standard error: $\frac{\sigma}{\sqrt{N}} = \frac{29.82}{\sqrt{4900}} = .42$

$$Z = \frac{25.09 - 25}{.42} = .21$$

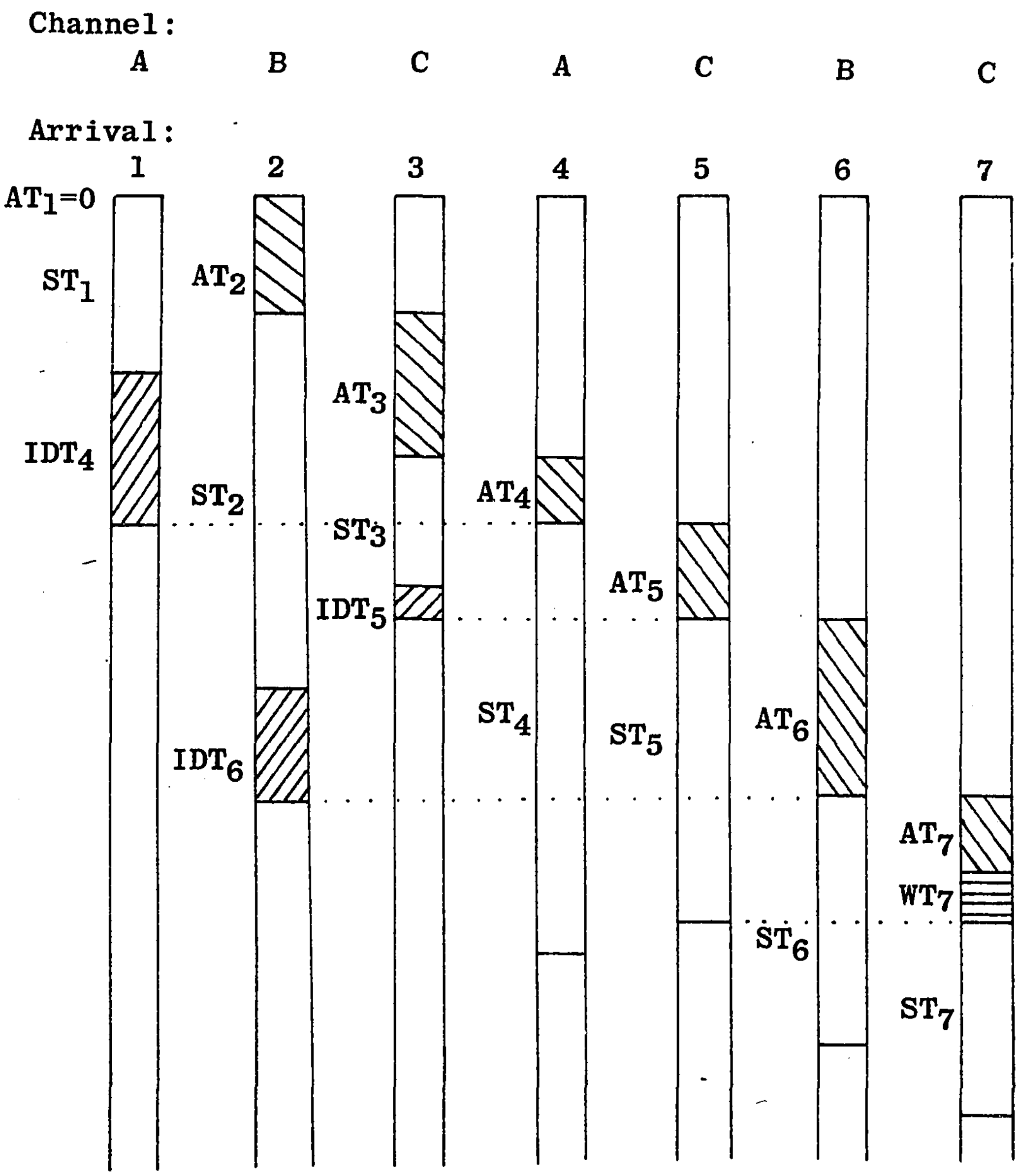
[5]The block diagram of this model is illustrated in figure (9) of the Appendix.

[6]The same logic applies to any number of channels.

Figure 7.

Graphical Representation of the Simulation Procedure

of a Multi-channel Model



The diagram represents a three-channel model with one waiting line and an admission policy of first come, first served. With N being the sample size, the ATi (i = 1, 2, . . ., N) are the interarrival intervals, the STi (k = 1, 2, . . ., N) the service time intervals, the IDTi (i = 1, 2, . . ., N), and the WTi (i = 1, 2, . . ., N) the waiting time intervals. As a unit arrives at the system, the service channels are checked to determine whether any one of them is unoccupied at the moment. If all three are occupied, waiting time occurs until one channel becomes vacant. When a channel becomes vacant before another unit arrives, idle time occurs until a unit arrives and enters the channel. Thus, in the above diagram, WT7 is a waiting time interval and IDT4, IDT5, and IDT6 are idle time intervals.

The analytical solution of the model is based on the following assumptions. First, there are k service stations in the system. Second, the input units arrive at the system in a poisson fashion with mean arrival rate, λ, equal to 1/5. And third, the service time intervals at each station are exponentially distributed with a mean service rate, μ, equal to 1/20. The analytical solution can be found by the following formula, where E(w) is the expected waiting time:[7]

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[7]Sasieni, Yaspan, Friedman, Operations Research - Methods and Problems (New York: Wiley, 1960), p. 138.

$$E(w) = \frac{\mu(\frac{\lambda}{\mu})^k}{(k-1)!\ (k\mu-\lambda)^2} \cdot$$

$$\frac{1}{\sum_{n=o}^{k-1} \frac{1}{n!} (\frac{\lambda}{\mu})^n + \frac{1}{k!} (\frac{\lambda}{\mu})^k (\frac{k\mu}{k\mu-\lambda})} \qquad (3f)$$

Again, 4,900 input units were simulated to pass through the system. However, for this model several runs were made, each with a different value for k. The simulation results and the corresponding analytical solutions for the runs are listed in the following table.

TABLE 1

VALUES OF THE EXPECTED WAITING TIME OF A MULTI-CHANNEL MODEL OBTAINED BOTH FROM ANALYTICAL SOLUTIONS AND SIMULATION RUNS

	Values of the Expected Waiting Time		
Number of Channels	Analytical Solutions	Simulation Results	Z Values
4	∞	Storage Overflow	-
5	11.081	11.07	0.41
6	3.632	3.75	0.94
7	1.159	1.09	1.32
8	0.398	0.39	0.30
9	0.124	0.12	0.31
10	0.038	0.03	1.30

In the above table, the storage overflow indicates that the value became very large and was assumed to be approaching infinity. The Z values are calculated by the formula $Z = \frac{\beta' - \beta}{\sigma_\beta}$, where β' is the analytical solution, β is the simulation result and σ_β is the standard error of β. A test of the tabulated values of Z indicates that the analytical solutions and the simulation results have no significant differences at the .05 probability level.

The results of these simulations would seem to indicate that the method selected for use is indeed appropriate for this study.

Summary

In this chapter, the average waiting times of input units to two basic waiting-line models were obtained by computer simulation and then compared to the analytical solutions of the same models. The conclusion reached on the basis of statistical analysis of the differences between the simulation results and analytical solutions was that the simulation method presented in Chapter II is adequate.

In the following chapter, the simulation method is used to construct models for which analytical solutions have not yet been found, but for which solutions can be obtained by employing the simulation procedure outlined in this chapter.

CHAPTER IV

ALTERNATIVE MODELS AND VALIDATION OF SIMULATION RESULTS

In this chapter, four simulation models are presented, each embodying some of the features which may be conceived to exist in a micro-economic system. These features either indicate that the distributions of the interarrival and service time intervals do not follow simple theoretical distributions or that they are dependently related to each other. Simulation runs of these four models are made with arbitrary parameter values to illustrate the kinds of information which can be obtained from the simulation results. Also, the method of determining the validity of these or similar simulation results are discussed at the end of this chapter.

Model A[1]

Model A is a single channel model with three service stations. The interarrival time intervals of the

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[1]The block diagram of this model is illustrated in figure (10) of the Appendix. All the assumptions made of this model and others to follow are arbitrary.

input units are assumed to follow an exponential distribution. The distributions of service time intervals for the three service stations, however, are assumed to be all different. These distributions are assumed as follows: station 1, normal distribution; station 2, exponential distribution; and station 3, lognormal distribution. Further, the mean of the arrival rate of the input units is assuming to vary depending upon the length of the time interval the previous unit stays in the system. If this time interval is longer than a specified period, the arrival rate will be decreased and conversely. This is one case of output-input feedback, in that the input rate is affected by the output rate of the system. The model can be modified by changing one or all of the following: the number of stations in the system, the probability distributions of the interarrival intervals and/or service time intervals, and the nature of the output-input feedbacks.

A simulation run with the following values for the parameters was performed to investigate the behavior of this hypothetical model of a micro-economic system: (1) the mean of the distribution of the service time intervals (i.e. normal distribution) at station 1 is set equal to ten minutes and the standard deviation of the distribution is assumed to equal to two minutes. (2) the mean and standard deviation of the distribution of the service time intervals (i.e., exponential) at station 2 are both set equal to fifteen minutes. (3) the mean of the distribution of the service time intervals (i.e., lognormal) at station 3 is assumed to equal to twenty minutes, and the standard deviation of the distribution is assumed to be five minutes.

And (4), it is assumed that if the total time that an input unit stays in the system is less than fifty minutes, the mean of the distribution of the interarrival intervals of the following units will be ten minutes. However, if the total time that an input unit stays in the system exceeds fifty minutes, the mean of the distribution of the interarrival intervals of the subsequent units will be increased to twenty minutes. In other words, the arrival rate changes from 1/10 unit per minute to 1/20 unit per minute, when it takes more than fifty minutes to complete the service on one unit.

The result of the simulation run shows that it will require 13,764 minutes to complete services on 100 input units. The average waiting time at station 1 is 6.52 minutes, at station 2, it is 18.94 minutes, and at station 3, it is 67.50 minutes. The total idle time at station 1 is 1,059 minutes, at station 2, 625 minutes, and at station 3, 87 minutes. Station 1 is engaged in service 48 per cent of the time, station 2, 70 per cent of the time, and station 3, 95 per cent of the time. These percentages show that the work load is not evenly distributed at the three stations; for example, station 1 is idle more than half of the time and station 3 is busy almost constantly.

An attempt was then made to find a more efficiently operating system by performing another simulation run with all assumptions remaining the same but with the following parameter changes: (1), the mean of the distribution of the service time intervals at station 1 is increased to fifteen minutes, and (2) the mean of the distribution of the service time intervals at station 3 is decreased to sixteen minutes. The assumption here is that by moving some of the facilities from station 1 to station 3, the service rate at station 3 will increase and the service rate at station 1 will decrease. The result of this second simulation run shows that it now takes only 10,094 minutes instead of 13,764 minutes to complete the services on the 100 input units and the work load is more evenly distributed, with station 1 engaged in service 74 per cent of the time, station 2,71 per cent of the time and station 3, 78 per cent of the time.

The above illustration shows that by moving facilities from the slack area to the congested area in a system, the total service time can be reduced, thus possibly decreasing the total cost of the services. The optimum distribution of the facilities among the various service stations in a system may be found by repeated simulation runs with different parameter values.

Model B[2]

Model B is a multi-channel model with N service stations in parallel arrangement. The mean arrival rate is assumed to be fluctuating according to a trigonometric function with its period equal to a specified interval. The admission policy is assumed to be first come, first served, and the service time intervals at each station are assumed to be exponentially distributed. This model may be modified by altering the admission policy and the distribution of the service time intervals at each station and may be further modified by assuming other kinds of cyclical curves that may trace out the change of the arrival rate. Also, Model A and Model B can be combined together to form a model with both series and parallel arranged service stations.

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[2]The block diagram of this model is illustrated in figure (11) of the Appendix.

A simulation run was made with the interarrival time intervals equal initially to twenty minutes. These intervals then fluctuated according to a trigonometric function with the period of each cycle equal to approximately twelve hours. At the peak of the cycle the interval is thirty minutes and at the trough, it equals ten minutes. In other words, the amplitude of the fluctuation is ten minutes both ways. The model is assumed to have five parallel arranged service stations and the mean of the distribution of the service time intervals of each station (i.e., exponential) is assumed to equal to eighty minutes. The results of the simulation run showed that about 8,187 minutes were required to complete services on the 100 input units. The average waiting time of the input units was 64.92 minutes, the average idle time for the five service stations was 295 minutes, and each station was engaged in service an average of 85 per cent of the time.

Now, suppose the management of this micro-economic system decides to open an additional service station. Another simulation run is made, but on this run there are six instead of five parallel arranged service stations. The results showed that now only 2,816 minutes are required to complete servicing the 100 input units. This resulted from drastic reduction of average waiting time from 64.92

to 14.03 minutes per input unit. However, the average idle time for each of the six service stations increased to 549 minutes, and each station is engaged in service an average of only 72 per cent of the time.

The above illustration shows that by increasing the number of service channels in a system, the total waiting time will be reduced while the total idle time will be increased. Given the unit cost of waiting time and idle time, the optimum number of service channels may be found by simulation runs with different numbers of channels.

Model C[3]

Model C has N parallel arranged main service stations and an extra ancillary service station. The input units arrive at the system in a poisson fashion and are admitted to the N main service stations whenever there is a vacancy. While occupying one of the main service stations, the unit first waits for the services of the ancillary service station. The length of the service time intervals at the ancillary service station follow an Erlang gamma distribution with the parameter K determined by the length of the waiting time of the preceding unit waiting

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[3]The block diagram of this model is illustrated in figure (12) of the Appendix.

to be served by the ancillary service station. If the waiting time of the preceding unit is longer than a specified interval, then K is reduced, and conversely. After being served by the ancillary service station, the unit remains in the main service station for a certain period of time, this latter period being assumed to follow a lognormal distribution. This model may be modified by changing the admission policies of both the main service stations and the ancillary service station, and by assuming alternative probability distributions for the interarrival intervals and for the service time intervals both at the main service stations and at the ancillary service station.

Suppose, for example, the model is a simulation of a simple school medical clinic which consists of a five-bed ward and a laboratory for testing. Assume further that (1) the time intervals between arrivals of the patients are exponentially distributed with mean and standard deviation equal to twenty-two hours, (2) the time intervals required to complete a laboratory test are also exponentially distributed, but with mean and standard deviation equal to three hours, and (3) the periods that the patients stay in the ward after having taken the tests follow a lognormal distribution with mean equal to twenty hours and standard deviation equal to four hours. Consider that a physician generally orders six tests per patient; however, when the waiting time between the ordering of a test and the order being carried out exceeds twenty hours, the physician orders only four tests per patient.

A simulation run with a sample of 100 patients passing through the ward gives the following results. The average waiting time of the patients to be accepted into the ward is 1.61 hours, the average waiting time between a test ordered and performed is 18.73 hours, and each bed in the ward is occupied an average of 46 per cent of the time and the laboratory is busy 69 per cent of the time.

Now, consider the following situation: the school authorities, in anticipation of a mild epidemic on the campus, desire to know how many patients this small clinic can service, knowing that the beds in the ward may be fully occupied all the time, and that the laboratory must be allowed some leisure time in order to prevent the technician from giving erroneous tests or inaccurate reports. Thus, another simulation run is made with the mean and standard deviation of the distribution of the interarrival intervals decreased to twenty hours, and the number of tests performed on each patient limited to the essential four. The result shows that the laboratory now is engaged in work 86 per cent of the time, and each bed in the ward is occupied an average of 99 per cent of the time. These results also indicate that with the capacity of the clinic and the service rate remaining the same, any further increase in the arrival rate will form a waiting line infinitely long, the consequence of which is that some of the patients have to be sent somewhere else for treatment.

The above illustration shows that the acceleration of the input rate to a system will increase its work load and thus may increase the total waiting time. One way to ease the situation is to reduce the number of services performed on each unit at the most congested point in the system. As with the preceding models, the optimum input rate in a particular situation may be found by repeated simulation runs with varying parameters.

Model D[4]

Model D is also a multi-channel model with input units arriving at the system in a poisson process. However, in this model when the unit is admitted into the system, the service time interval is composed of two parts. The first part is assumed to be in Erlang gamma distribution with the value of the parameter K varying according to a

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[4]The block diagram of this model is illustrated in figure (13) of the Appendix.

known probability distribution, and the second part is assumed to be lognormally distributed with mean and standard deviation functionally related to the length of the time interval of the first part. If the first part of the interval is longer than a given length, then the mean and standard deviation of the distribution of the second part of the interval will be longer and conversely. Model modification may be achieved by altering the distribution of the interarrival intervals, the distributions of the two parts of the service time intervals, and the functional relationship between the first part and the second part of the interval.

This model may also be considered as a simple hospital model. The period during which the patients stay in the ward may be broken into two intervals; the first interval is the diagnostic period, the second interval the convalescent period. Assume that there are five beds in the ward, and that the patients arrive at the ward in intervals with a mean and a standard deviation both equal to twenty hours. Further, assume that 60 per cent of the patients need to take six different laboratory tests or preliminary examinations before the completion of the diagnostic period, and the other forty per cent need take only three. If the diagnostic period is longer than 100 hours, then the convalescent period of the patient will be a variate of a lognormal distribution with mean equal to 100 hours and standard deviation equal to twenty hours. Otherwise, the mean and standard deviation are equal to fifty and ten hours.

The results of a simulation run with a sample of 100 patients passing through the ward of this model show that the average waiting time of the patients to get into the ward is 5.11 hours, and the average time interval that each patient stays in the ward is 66.33 hours. Also, each bed in the ward is occupied an average of 63 per cent of the time.

Now, suppose the community which the hospital serves is growing, and the absolute number of patients increases with the increase of the population. The model can be altered to account for this increase in the number of patients by decreasing the mean and standard deviation of the distribution of the interarrival intervals to fifteen hours. A question is raised as to how the situation can be met without increasing the existing facilities of the hospital. One solution is to shorten the convalescent periods of the patients staying in the hospital. To examine this possibility, another simulation run was made with the mean of the distribution of the convalescent periods reduced to eighty hours for those whose diagnostic periods exceeded

100 hours, and forty hours for those whose diagnostic period did not exceed 100 hours. The results show that the average waiting time is now 6.93 hours, the average time that each patient stays in the ward, 56.05 hours, and the beds in the ward are occupied by the patients 71 per cent of the time.

The results of these simulation runs clearly show that the increase of the work load due to the increase of the input rate to a system may be balanced by shortening the length of the service time spent on each unit. The optimum balance of the input rate and the service rate of a particular system may be found for this model by simulation runs with different sets of input rates and service rates.

The models presented above are only intended as illustrations. Model A indicates the effect of rearranging facilities within a system. In Model B, the effect of changing the number of parallel arranged service stations is illustrated. In the third model, Model C, the effect of altering the decision rules regarding admission policy and requests for laboratory services in a hypothetical school clinic is evaluated, and in Model D, the effect of changing the discharge policy in a simple hospital system to meet the changing demand is studied. There is, however, other information which can be obtained by experimenting with the models. For example, the distribution of the waiting time or idle time may be approximated by smoothing the histograms which can be generated from information printed out by the computer for each individual input unit. The limitation of time and money has restricted this investigation to the study of the behavior of these fictitious models, but the analysis has indicated that the same method and procedure can be used to simulate similar situations as long as they are correctly defined.

Validation of Simulation Results

After a simulation model has been constructed and the simulation runs have been made, a question immediately arises: how valid are these simulation results?

If a computer simulation is made of an existing system for the purpose of explaining and predicting the behavior of that system, the results of the simulation runs can be compared continuously to sample observations made on the existing system, and in that way insure the validity of the simulation model. But if the computer simulation is for the purpose of helping design a new system and actual observations cannot be made, then the validity of the model will have to depend on how realistic the basic assumptions are and whether the relationships derived from these basic assumptions have been logically deduced correctly.

A simulation model of a micro-economic system when first programmed may be too simple an extraction from reality or may have errors in its basic assumptions. For example, a common error occurs when an interdependent scheduling process is applied to the system, that is, when the arrivals are scheduled in such a way that the interarrival time intervals fluctuate in phase with the fluctuations of the service time intervals. When the service time becomes longer, the arrivals will be scheduled to arrive at the system in longer time intervals, and when the service time becomes shorter the arrivals will be scheduled to arrive at the system more frequently. If this kind of interdependence between the time variates does exist in a real situation, but is not considered while constructing the simulation model, then even though the distributions of both the interarrival time intervals and the service time intervals are correctly described by the simulation model, a large discrepancy may occur between the observed waiting time idle time of the real situation as compared to the result of the simulation runs. But as the subprograms and the main program can be modified easily, and the parameters such as the means and variances of the distributions of the variables are set for each simulation run, it is possible to constantly review and improve the simulation model and keep it from deviating from reality.

Summary

Four waiting-line models of assumed situations are presented in this chapter, and the results of the simulation runs are obtained and analyzed.

The major purposes for which these models were derived are: (1) to show that once the relationships between all the elementary parts of a micro-economic system are correctly described, they can be simulated on a computer, and (2) to show that the results of the simulation runs will give an indication of the behavior and input-output relationships of the system. Further, it was observed that the validity of the simulation results can be determined either by comparing the simulation results with the observations of an existing real system or, if the real system does not exist, to check the logical consistency of the basic assumptions of the simulation model.

In the following chapter, the significance of these simulation results to economics is presented and discussed.

CHAPTER V

SIGNIFICANCE OF THE COMPUTER SIMULATION TO ECONOMICS

This chapter consists of three distinct yet related topics. First, after briefly reviewing the simulation procedures presented in the preceding chapters, the usefulness of a simulated micro-economic system in explaining and predicting the behavior of the real system is discussed. Second, the use of the simulation results as a basis for making rational decisions is illustrated. And last, the results of extensive computer runs on the single-channel and multi-channel models presented in Chapter III are analyzed in an attempt to draw conclusions which would apply to other micro-economic systems with similar or more complex structures.

A Simulated Micro-economic System

A micro-economic system usually embodies processes which are stochastic in nature, such as the customers arriving at a store, the patients arriving at a hospital, the inputs of raw materials in a production process, or the time required to complete a service or convert raw materials into a finished product. According to the simulation method discussed previously, these stochastic processes can be described by probability distributions which can then be simulated by computer subprograms written in FORTRAN. Additionally, the structure of the system and the flows inside the system can be described by a block diagram, which is then translated into a main FORTRAN program.

In this investigation the key stochastic processes of interest are the arrival, service, waiting, and idle times at service points in the modeled economic environments. Subprograms were written to simulate the variations of these time intervals. In addition, for each of the models considered, main programs were constructed to describe the time dimensional flows in these systems. The computer runs on these models then produced sets of data showing the amounts of service, idle, and waiting time at service stations in the system, given different sets of assumptions and parameter values.

To indicate the relevance of this procedure to economic investigations, it is necessary only to consider one of the major objectives of these investigations, namely, prediction. An inherent characteristic of a micro-economic system, or for that matter any economic system, is its dynamism: The system is constantly changing. If the kinds and magnitudes of these changes could be anticipated with certainty, prediction would be precise; unfortunately, this is seldom the case. An alternative to certain knowledge of these changes is to construct a set of possible changes, and then analyze the system in light of these assumed changes. The simulation method presented in this study affords the researcher a means by which a modeled environment of an economic system can be studied under different sets of assumptions. To illustrate, the models studied in this thesis represented certain micro-economic systems, from which policy decisions were to be made. These models were confronted by hypothesized changes in order to determine the effects on the behavior of the systems. The parameter values for these models, which include the number of service channels and stations, the means and standard deviations of the probability distributions of the interarrival time intervals and service time intervals, can be set at the beginning of each simulation run. Therefore, if changes are desired, the only requirement is to change the parameter values before making the simulation run. Other changes, such as altering the assumptions about the kinds of probability distributions used, can be effected by calling for different subprograms. Also, modification of the simulation model itself, achieved by altering the arrangement of the facilities or decision rules, can be made by reconstructing the main program. After all the desired changes have been made, the simulation run will then show how these changes will affect the service time, the waiting time, and the idle time.

Further, the sensitivity and the stability of the system can be determined by inspecting the changes in the waiting time and/or idle time in response to the changes of the parameter values.

Sensitivity of a System

Sensitivity of a system is defined as the change of the behavior of the system in response to a gradual change in the value of only one parameter at a time. The following is an illustration:

A single-station waiting-line model is constructed with the following assumptions:[1] (1) 900 input units pass through the system. The service discipline is first come, first served; (2) the interarrival time intervals of the input units are exponentially distributed with mean and

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[1]All the assumptions made of the models used in this investigation are arbitrary. They are only for illustrative purposes.

standard deviation both equal to fifteen minutes; (3) the service time intervals are normally distributed with varying mean and standard deviation. First, the mean is set at ten minutes and the standard deviation is varied from eight minutes to one minute. And then, the mean is reduced to nine minutes and the standard deviation varied from nine minutes to five minutes. Table 2 shows the results of these simulation runs. It indicates that by reducing the mean of the distribution of the service time intervals from ten minutes to nine minutes, while holding the standard deviation constant at eight minutes, the total waiting time will decrease from 14,924.53 minutes to 11,159.14 minutes. However, if the mean is held constant at ten minutes, then it is necessary to reduce the standard deviation from eight minutes to five minutes in order to reduce the total waiting time to 11,525.82 minutes.

Stability of a System

Stability of a system indicates whether the state of the system will always be the same or will change continuously as time passes. In a system with waiting lines, the state of the system may be classified as empty, transient, stable or explosive. When a system has no input unit in it, it is in an empty state. During the period

TABLE 2

RESULTS OF SIMULATION RUNS SHOWING THAT TOTAL WAITING TIME CAN BE REDUCED EITHER BY DECREASING THE MEAN OR THE STANDARD DEVIATION OF THE DISTRIBUTION OF THE SERVICE TIME INTERVALS

Total Number of Input Units	Interarrival Time Intervals*	Service Time Intervals**		Total Waiting Time	Service Time Intervals**		Total Waiting Time
	Mean and S.D.	Mean	S.D.		Mean	S.D.	
900	15 min.	10 min.	8 min.	14,924.53 min.	-	-	-
900	15	10	7	13,683.44	-	-	-
900	15	10	6	12,554.85	9 min.	9 min.	12,387.47 min.
900	15	10	5	11,525.82	9	8	11,159.14
900	15	10	4	10,650.62	9	7	10,030.73
900	15	10	3	9,868.00	9	6	8,984.74
900	15	10	2	9,280.88	-	-	-
900	15	10	1	8,804.33	9	5	8,098.24

* The distribution is assumed to be an exponential distribution.
** The distribution is assumed to be a normal distribution (positive values only).

when input units begin to arrive at the system, gradually filling it up, the system is said to be in a transient state. After the system reaches the point where input units are passing through and leaving, the system may reach either a stable state; that is, one where the probability that the number of units waiting to enter at any instant remains the same as time passes, or the system may reach an explosive state, in which case the waiting line increases infinitely with time.[2] Experimenting with a simulation model as described above will indicate which set of parameter values will make the system stable and which set of parameter values will cause the waiting lines to become infinitely long as time passes.

Management Decision Making

Given the ability to explain the behavior of a micro-economic system and to predict the change of its behavior resulting from modifications of some of its initial conditions, management of the system will then have a basis for making rational decisions in accordance with some given set of policies.

Assume that the policy is to minimize the total

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[2]M. Sasieni, et al, Operations Research - Methods and Problems (New York: John Wiley and Sons, Inc., 1960), p. 125.

cost of waiting time and idle time during the process of rendering a given number of services. The following are illustrations of the kind of information which the simulation runs can provide and which management may need to know in order to make rational decisions.

(1) Given the input and service rates of a system, management may want to know the optimum number of service channels there should be in the system. The simulation runs will give the total waiting time and idle time occurred while rendering a given number of services. And if the unit costs of waiting time and idle time can be estimated, then the product of the total waiting time and the unit cost of waiting time is the total cost of waiting time and the product of the total idle time and the unit cost of idle time is the total cost of idle time. Compare the total cost of both waiting time and idle time occurred with various number of channels. The number of channels which give the minimum total cost is the optimum number. The following serves as an illustration:

A multi-channel model is constructed with the following assumptions: (a) The interarrival time intervals follow an exponential distribution with mean and standard deviation both equal to twenty minutes. (b) The service time intervals at each channel are exponentially distributed with mean equal to eighty minutes. (c) The number of channels vary from four to eight. (d) Both the unit costs of waiting time and idle time are $.01 per minute.

Table 3 shows the results of the simulation runs in which 900 units pass through the model. The results indicate that with five channels the total cost of waiting time and idle time is at the minimum, which is $3,825. The rightmost column of Table 3 shows that if the standard deviations of both distributions are zero, meaning that the input and service rates are both constant, then the total cost of waiting and idle time will be zero when there are only four channels.

(2) Given the service rate of a system, management may want to know the input rate which will minimize the total cost of waiting time and idle time. The following is such an illustration.

A single-channel three-station model is constructed with the following assumptions: (a) the service time intervals at station 1 are normally distributed with a mean of ten minutes and standard deviation of two minutes; (b) the service time intervals at station 2 are exponentially distributed with mean and standard deviation both equal to fifteen minutes; (c) the service time intervals at station 3 are lognormally distributed with mean equal to twenty

TABLE 3

RESULTS OF SIMULATION RUNS INDICATING THE OPTIMUM NUMBER OF SERVICE CHANNELS, GIVEN THE INPUT AND SERVICE RATES

Total Number of Input Units	Interarrival Time Intervals*	Service Time Intervals at Each Channel*	Number of Channels	Total Cost of Waiting Time and Idle Time**	Compared to the Total Cost of Constant Interarrival (20 min.) and Service Time Intervals (80 min.)
	Mean and S.D.	Mean and S.D.			
900	20 min.	80 min.	4	$5,805	$ 0
900	20	80	5	3,825	1,800
900	20	80	6	4,590	3,600
900	20	80	7	6,138	5,400
900	20	80	8	7,938	7,200

* Both the interarrival time intervals and the service time intervals are assumed to be exponentially distributed.

** Both the unit costs of waiting time and idle time are assumed to be equal to $.01 per minute.

minutes and standard deviation equal to five minutes; (d) the interarrival time intervals are exponentially distributed with mean and standard deviation varying from ten minutes to twenty-eight minutes, with each increment equal to two minutes; (e) both the unit costs of waiting time and idle time are equal to $.01.

Table 4 shows that the results of simulation runs in which 900 units pass through the model. The results indicate that when the mean and standard deviation are equal to twenty-six minutes, the total cost of waiting time and idle time will be the least, which is $72,090.

Further Results of the Simulation Runs

In an attempt to draw conclusions which would apply to micro-economic systems with similar or more complex structures, additional simulation runs on the two models described in Chapter III are made. The results of these simulation runs are presented below.

(1) Table 5 shows the results of simulation runs in which 900 units pass through a single-station model with the following assumptions: (a) the input units will form a single waiting line. The service discipline is first come, first served; (b) the interarrival time intervals are exponentially distributed with mean and standard deviation

TABLE 4

RESULTS OF SIMULATION RUNS INDICATING THE OPTIMUM INPUT RATE, GIVEN THE SERVICE RATES

Total number of Input Units	Service Time Intervals: Service Sta. 1*		2**		3***		Interarrival Time Intervals****	Total Cost of Waiting Time and Idle Time*****
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean and S.D.	
900	10min.	2min.	15min.	15min.	20min.	5min.	10 min.	$49,050
900	10	2	15	15	20	5	12	39,537
900	10	2	15	15	20	5	14	30,096
900	10	2	15	15	20	5	16	21,240
900	10	2	15	15	20	5	18	14,112
900	10	2	15	15	20	5	20	99,090
900	10	2	15	15	20	5	22	81,180
900	10	2	15	15	20	5	24	74,250
900	10	2	15	15	20	5	26	72,090
900	10	2	15	15	20	5	28	73,440

* The Service time intervals at station 1 are assumed to be normally distributed (positive values only).

** The service time intervals at station 2 are assumed to be exponentially distributed.

*** The service time intervals at station 3 are assumed to be lognormally distributed.

**** The interarrival time intervals are assumed to be exponentially distributed.

***** Both the unit costs of waiting time and idle time are assumed to be equal to $.01 per minute.

TABLE 5

RESULTS OF SIMULATION RUNS INDICATING, OTHER THINGS BEING EQUAL, THE GREATER THE VARIANCE OF THE SERVICE RATE, THE LONGER WILL BE THE TOTAL WAITING TIME

Total Number of Input Units	Interarrival Time Intervals*	Service Time Intervals**		Total Waiting Time	Total Idle Time	Total Waiting Time and Idle Time
	Mean and S.D.	Mean	S.D.			
900	15 min.	10 min.	1 min.	8804.33 min.	5260.86 min.	14065.19 min.
900	15	10	2	9280.88	5240.24	14521.12
900	15	10	3	9868.00	5219.61	15087.61
900	15	10	4	10650.62	5198.99	15849.61
900	15	10	5	11525.82	5178.36	16704.18
900	15	10	6	12554.85	5157.73	17712.58
900	15	10	7	13683.44	5137.11	18820.55
900	15	10	8	14924.53	5116.48	20041.01
900	15	10	9	16307.00	5095.86	21402.86
900	15	10	10	17831.08	5075.23	22906.31

* The distribution is assumed to be an exponential distribution.
** The distribution is assumed to be a normal distribution (positive values only).

equal to fifteen minutes; (c) the service time intervals are normally distributed with mean equal to ten minutes, but the standard deviation increases from one minute to ten minutes in increments of one minute. The results indicate, other things being constant, as the standard deviation of the distribution of the service time intervals is increasing, the total waiting time is steadily increased.

(2) Table 6 shows the results of simulation runs on the same model only this time the assumptions of the interarrival time and service time intervals are different: (a) the service time intervals are exponentially distributed with mean and standard deviation both equal to ten minutes; (b) the interarrival time intervals are normally distributed with a mean of twelve minutes and standard deviation varying from one minute to nine minutes. The results indicate, other things being equal, as the standard deviation of the distribution of the interarrival time intervals is increasing, the total waiting time is steadily increased. Thus, it can be concluded on the basis of the above evidence that when the means of the distributions of interarrival time and service time intervals remain the same, the greater are the variances of the distributions, the larger will be the total waiting time. Further, the simulation results indicate that variation in the interarrival time or service time intervals

TABLE 6

RESULTS OF SIMULATION RUNS INDICATING, OTHER THINGS BEING EQUAL, THE GREATER THE VARIANCE OF THE INPUT RATE, THE LONGER WILL BE THE TOTAL WAITING TIME

Total Number of Input Units	Interarrival Time Intervals*		Service Time Intervals**	Total Waiting Time	Total Idle Time	Total Waiting Time and Idle Time
	Mean	S.D.	Mean and S. D.			
900	12 min.	1 min.	10 min.	16,360.75 min.	1,660.10 min.	18,020.85min.
900	12	2	10	17,397.12	1,645.00	19,042.12
900	12	3	10	18,948.72	1,629.90	20,578.62
900	12	4	10	20,711.42	1,614.80	22,326.22
900	12	5	10	22,729.60	1,604.01	24,333.61
900	12	6	10	25,319.01	1,594.44	26,913.45
900	12	7	10	28,665.97	1,584.88	30,250.85
900	12	8	10	32,637.53	1,575.32	34,212.85
900	12	9	10	37,688.07	1,565.76	39,253.83

* The distribution is assumed to be a normal distribution (positive values only).
** The distribution is assumed to be an exponential distribution.

may sometime cause a reduction in the total idle time.

An explanation of the above findings is that variation in the input or service rates will often result in longer waiting time for some input units. And these longer waiting times will tend to make the waiting time of succeeding units even longer; thus, the total waiting time will increase. Now, if the waiting time has increased to the point where a waiting line is formed, which will reduce the probability of having the service station become idle, then the total idle time will decrease. However, if a waiting line is not formed, variation in the input and/or service rates will cause both the waiting time and the idle time to increase. These findings indicate clearly that it would be advantageous to reduce the variances of both service and input rates, since this would not only reduce the total waiting time but would also avoid the problem of service stations being idle at one time and busy at the next.

(3) Table 7 shows the results of simulation runs in which 900 units pass through a single-channel three-station model with the following assumptions: (a) the inter-arrival time intervals are exponentially distributed with mean and standard deviation equal to fifteen minutes; (b) the service time intervals at all three stations follow exponential distributions but with different means and

TABLE 7

RESULTS OF SIMULATION RUNS INDICATING, IN A SINGLE CHANNEL MODEL, TOTAL WAITING TIME WILL BE THE LEAST WHEN THE SERVICE RATE IS PROGRESSIVELY INCREASING FROM THE FIRST SERVICE STATION TO THE LAST SERVICE STATION

Total Number of Input Units	Mean and Standard Deviation of the Distribution of the Interarrival Time Intervals*	Mean and Standard Deviation of the Distributions of the Service Time Intervals*			Total Waiting Time	Total Idle Time	Total Waiting Time and Idle Time
		First Service Station	Second Service Station	Third Service Station			
900	15 min.	9 min.	10 min.	11 min.	49240.03	15200.27	64440.30
900	15	10	10	10	44884.84	15237.46	60122.30
900	15	10.2	10	9.8	44440.27	15245.08	59685.35
900	15	10.3	10	9.7	44279.00	15248.89	59527.89
900	15	10.4	10	9.6	44189.09	15252.70	59435.79
900	15	10.5	10	9.5	44169.83	15256.52	59426.35
900	15	10.6	10	9.4	44259.65	15260.33	59519.98

* The distributions are assumed to be exponential distributions.

standard deviations; at station 2, the mean and standard deviation are equal to ten minutes; at station 1, they increase from nine minutes to 10.6 minutes; while at station 3, they decrease from eleven minutes to 9.4 minutes. The results show that the total waiting time and idle time is the least when the mean and standard deviation of the distribution of the service time intervals at station 1 is equal to 10.5 minutes; at station 2, ten minutes; and at station 3, 9.5 minutes.

The following conclusion can be drawn from these results: In a single-channel multi-station model minimum total cost can be obtained, not when the mean of the distributions of the service time intervals are exactly equal among the service stations, but rather when the mean is gradually decreasing. In other words, the mean of the service rate becomes progressively larger from the first station to the last one.

The explanation of this is that once waiting time occurs at the stations in the front and a waiting line has been formed, idle time tends to decrease at these stations and the units will be passed on to the stations in the rear with increasing speed; thus waiting time tends to be longer at the stations in the rear and, consequently, will require a faster service rate at these stations to reduce it.

(4) Table 8 shows the results of simulation runs in which 900 units pass through a three-channel model with the following assumptions: (a) when the input units arrive at the system, if no channel is empty at the moment, they will form a single waiting line, but if more than one channel is empty, a priority rule will be imposed; that is, the unit will enter the system by the order of channel 1, 2, and 3 always; (b) the interarrival time intervals are exponentially distributed with mean and standard deviation both equal to 25 minutes; (c) the service time intervals at all three channels are normally distributed with the same standard deviation of 20 minutes, but different means: at channel 2, the mean is 60 minutes; at channel 1, first, the mean decreases from 59 minutes to 55 minutes, then it increases from 61 minutes to 65 minutes; at channel 3, first the mean increases from 61 to 55 minutes, and then decreases from 59 minutes to 55 minutes. The results show that the total waiting time is at the minimum of 24,957 minutes when the mean at channel 1 is 56 minutes; at channel 2, 60 minutes; and at channel 3, 64 minutes. Also they show that when channel 1 has faster service rate than channel 3, the total waiting time will be less than when the service rate is faster at channel 3.

The above finding indicates that when the service

TABLE 8

SIMULATION RESULTS INDICATING THAT WHEN THE SERVICE RATES ARE NOT EQUAL IN A MULTI-CHANNEL MODEL, PRIORITY ASSIGNMENT SHOULD BE GIVEN TO THOSE CHANNELS HAVING FASTER SERVICE RATES

Total Number of Input Units	Interarrival Time Intervals*	Service Time Intervals** (S.D. = 20 min.)			Total Waiting Time
	Mean and S.D.	Means Channel 1	Channel 2	Channel 3	
900	25 min.	59 min.	60 min.	61 min.	28,179 min.
900	25	58	60	62	27,378
900	25	57	60	63	26,559
900	25	56	60	64	24,957***
900	25	55	60	65	26,892
900	25	61	60	59	29,214
900	25	62	60	58	28,188
900	25	63	60	57	27,261
900	25	64	60	56	28,485
900	25	65	60	55	29,673

* The distribution is assumed to be exponential distribution.

** The distributions are assumed to be normal distributions (positive values only).

*** The 95 per cent confidence interval for the minimum average waiting time is 27.73 ± .89 minutes (S.D. is 13.65 min.), and for the two neighboring points are 29.51 ± .88 and 29.88 ± .88 minutes (S.D. is 13.52 min.). Since the first interval does not overlap with the latter two intervals, it can be said that when the mean service rate at channel 1 is 56 minutes, channel 2, 60 minutes and channel 3, 64 minutes, probability is .95 that waiting time is at the minimum.

rates at the different channels of a multi-channel model are not equal, priority assignment should be given to those channels having faster service rates in order to have smaller total waiting time.

(5) Table 9 shows the results of the simulation runs in which 900 units pass through a multi-channel model under the following hypothetical conditions: (a) the interarrival time intervals are exponentially distributed with mean and standard deviation both equal to 20 minutes; (b) the service time intervals at each channel are exponentially distributed with mean and standard deviation equal to 80 minutes; (c) the unit cost of idle time decreases from $.004 to $.001 and the unit cost of waiting time increases from $.001 to $.005. Thus, the ratio of the unit cost of idle time and waiting time decreases from 4 to 1/5. The results of simulation runs show that the optimum number of channels, when the ratio is 4 or 2, is five; when the ratio is 1 or 1/2, the optimum number is six; and when the ratio is 1/3, 1/4, or 1/5, the optimum number is seven.

The following conclusion can be drawn from the above findings. When the ratio of the unit cost of the idle time to the unit cost of waiting time is decreasing, the input rate should be reduced and/or the service rate should be increased continuously in order to minimize the total cost of

TABLE 9

RESULTS OF SIMULATION RUNS INDICATING THAT THE OPTIMUM NUMBER OF CHANNELS INCREASES AS THE RATIO OF UNIT COST OF IDLE TIME AND WAITING TIME DECREASES

Total Number of Input Units	Interarrival Time Intervals*	Service Time Intervals*	Unit Cost ($/min.) Idle Time	Unit Cost ($/min.) Waiting Time	Ratio	Minimum Total Cost of Waiting Time and Idle Time	Optimum Number of Channels
	Mean and S.D.	Mean and S.D.					
900	20 min.	80 min.	$.004	$.001	4	$11,682	5
900	20	80	.002	.001	2	10,314	5
900	20	80	.001	.001	1	8,208	6
900	20	80	.001	.002	1/2	5,112	6
900	20	80	.001	.003	1/3	5,598	7
900	20	80	.001	.004	1/4	5,985	7
900	20	80	.001	.005	1/5	6,372	7

* The distributions are assumed to be exponential distributions.

waiting time and idle time. In a multi-channel model, if this ratio has decreased to a point where the total cost of waiting time and idle time with the existing number of channels exceeds that with an additional channel put into the system, then the number of channels should be increased. The converse of the above situation is also true.

Summary

This chapter has shown the usefulness of the computer simulation method in explaining and predicting the behavior of micro-economic systems. Thus, when this method is used, it will provide valuable information for management decision making.

Using waiting-line models as illustrations, and assuming that the objective of management is to minimize the total cost of waiting time and idle time in rendering a given number of services, the following conclusions are drawn from the results of the simulation runs. (1) Other things being equal, the larger the variances of the service rate and/or the input rate, the longer will be the total waiting time provided that the rates are independent of each other. Total idle time will be increased when no waiting line has been formed. It may be reduced, however, when a waiting line has been formed and maintained. (2) In a single-channel multi-station model, total waiting time may be reduced when the service rate is gradually increasing from the stations in the front to the stations in the rear. (3) If the service rates at the different channels of a multi-channel model are not equal, total waiting time may be reduced when priority are assigned to those channels having faster service rate. And, (4) when the ratio of the unit cost of the idle time to the unit cost of the waiting time is decreasing, the service rate should be increased and/or the input rate should be decreased in order to minimize the total cost of waiting time and idle time. In a multi-channel model, in addition to the above, the number of channels should be increased provided that this ratio has increased enough so that the total cost incurred with the existing number of channels exceeds the total cost which would be incurred when an additional channel is put into the system.

These are the general conclusions about the behavior of micro-economic systems with the service stations either series or parallel arranged. The optimum combination of input and service rates for a particular system, however, must be determined by the simulation runs with a specific computer program written for that system alone.

In the next and concluding chapter, the application of the simulation method used throughout this investigation on other aspects of a micro-economic system is discussed.

CHAPTER VI

CONCLUSION

It was indicated in the introductory chapter to this thesis the limitations of using verbal descriptions and mathematical equations to describe the structure and interrelationships in micro-economic systems. It was also indicated that the major purpose of this thesis was to demonstrate a method, namely, computer simulation, which could in many instances provide a more flexible and usable means of analyzing the behavior of such systems, particularly those which when expressed as a system of mathematical equations were too complex to admit of analytical solution.

It is important to note, however, computer simulation does not necessarily exclude verbal descriptions and mathematical equations; to the contrary, they may well supplement each other. For instance, accurate and detailed verbal description of a system as well as established mathematical solutions of some of the relationships in the system will make computer simulation a simpler task and may well increase the validity of its results. On the other hand, the results of computer simulation may direct the effort to finding original or additional mathematical solutions of these relationships.

The simulation method introduced in this study consists of two basic parts: (1) the stochastic processes of the elementary parts are simulated by computer (FORTRAN) subprograms, and (2) the flows in the system among the elementary parts are described by a block diagram which is then translated into a main program. The latter part thus represents the simulated system. The validity of this method is insured by comparing the simulation results with the established analytical solutions of two basic single-channel and multi-channel models.

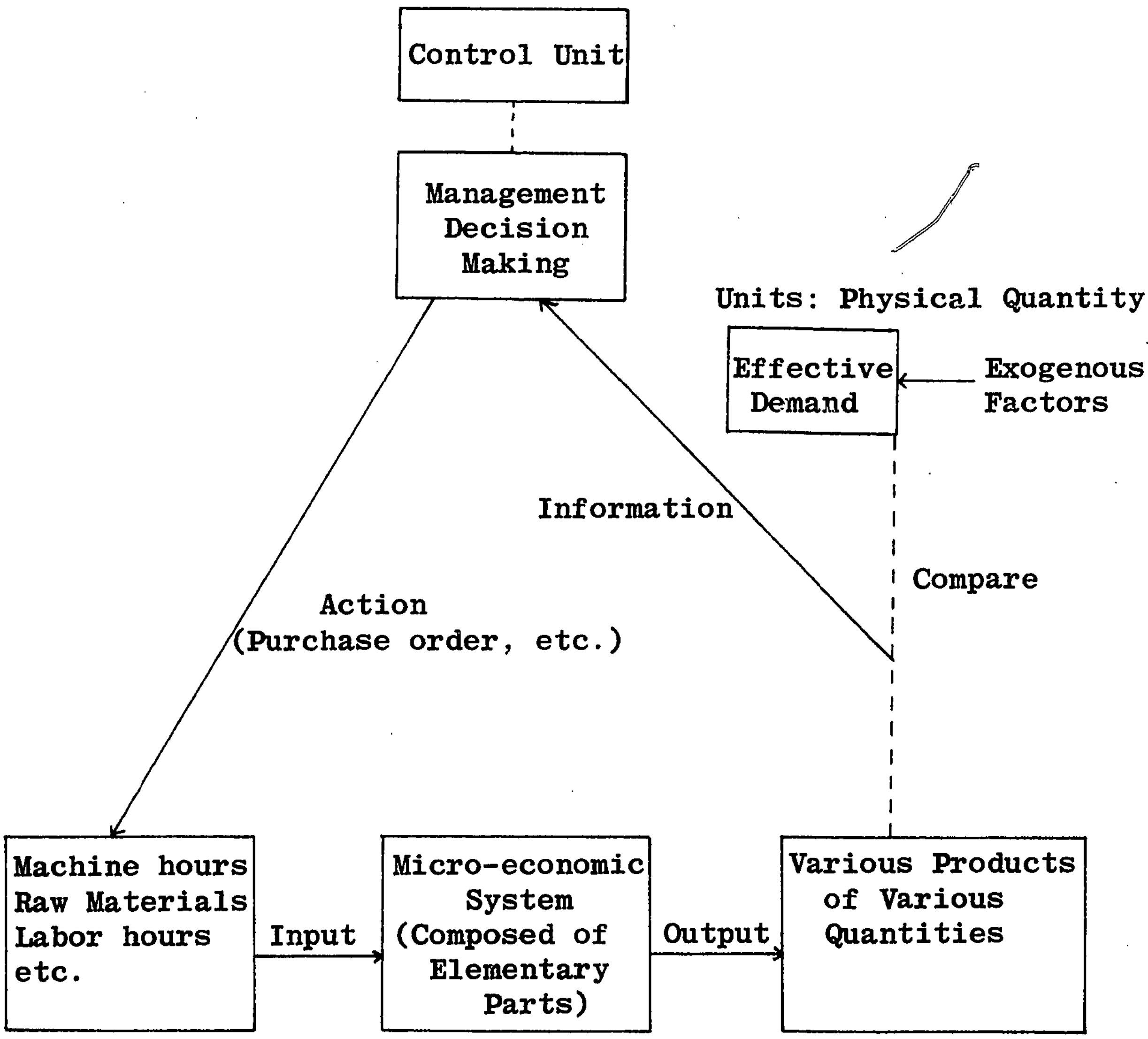
In addition to the two basic waiting line models, four more fictitious models were presented to illustrate the simulation of the stochastic relationships of arrival and waiting time of the input units and service and idle time of the service facilities in micro-economic systems. It is, however, only one type of micro-economic systems which has been chosen as a demonstration of the simulation method. Other types such as changes in inventory level, movement in production assembly lines, the choice of product mix, etc., also can be simulated on an electronic computer using the same technique. The following diagram, Figure (8), indicates such an example.

In the diagram, the inputs to the system are raw materials, labor hours, machine hours, etc., and the outputs are the various products which the system is organized to produce. The conversion ratios between the inputs and the outputs, or in economic terms the production functions, may vary according to certain probability distributions. Simulation runs can be made on the computer to simulate these production processes. The quantities of the various products obtained as the result of these simulation runs can be compared to the effective demands for these products. This information may then be passed on to the control unit which will make proper adjustment either of the quantities of the inputs or of the conversion ratio. This having been accomplished, another simulation is run in a further attempt to establish the optimum level of production.

It is the view of the writer that in near future, computer simulation will be applied extensively to economic systems of all kind. It will help management in controlling and improving existing systems, and in planning new systems.

Figure 8

Arrow Diagram of the Model of A Production System with Raw Materials, Labor Hours, and Machine Hours as its Inputs, and Products as its Outputs



Simulate Conversion Relationships (Production Functions at every Stage)

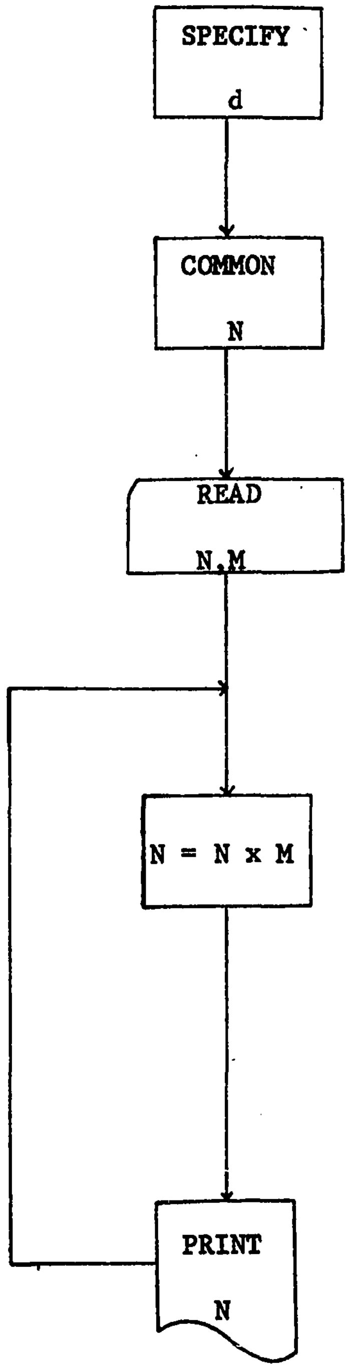
APPENDIXES

APPENDIX A

BLOCK DIAGRAMS

Appendix A contains the block diagrams of the six waiting-line models and the generation of the various probability distributions presented in this thesis.

Appendix Figure 1. Generation of Random Numbers



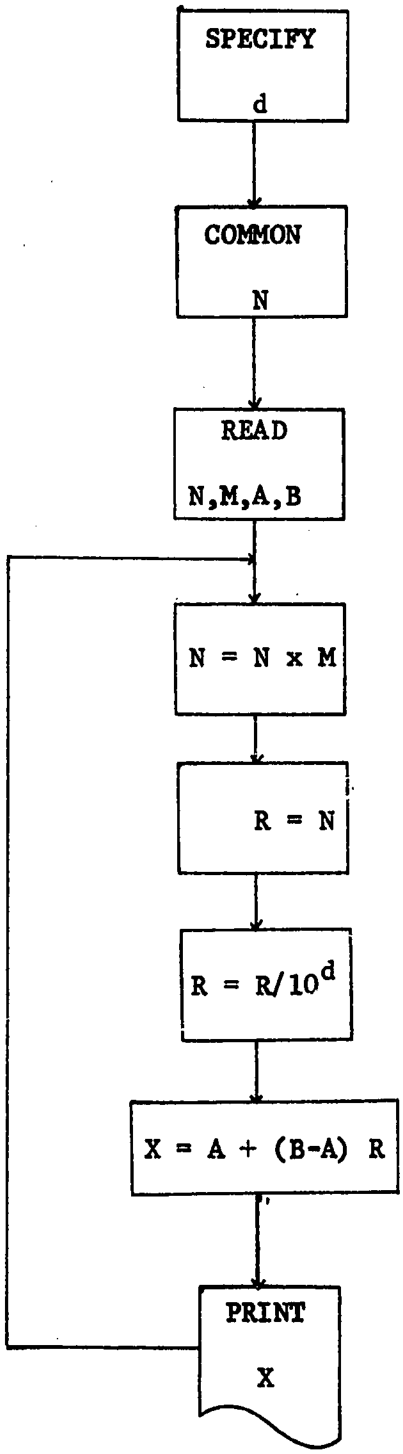
d is the number of digits contained in the random number variate.

N is the random number variate.

M is the constant multiplier.

Only the lower d-digits of N are retained.

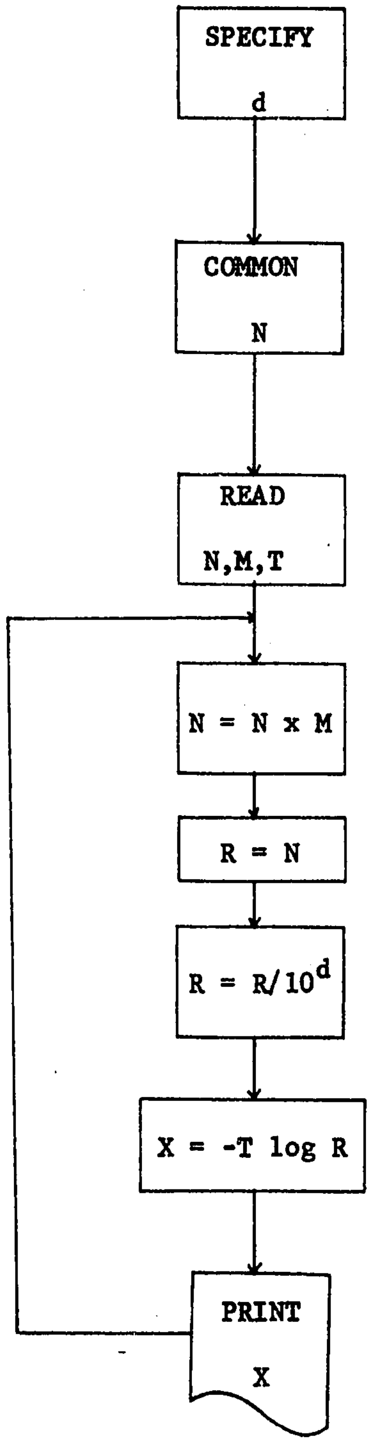
Appendix Figure 2. Generation of Uniformly distributed Variate



A is the lower limit and B is the upper limit of a uniformly distributed variate.

X is the uniformly distributed variate.

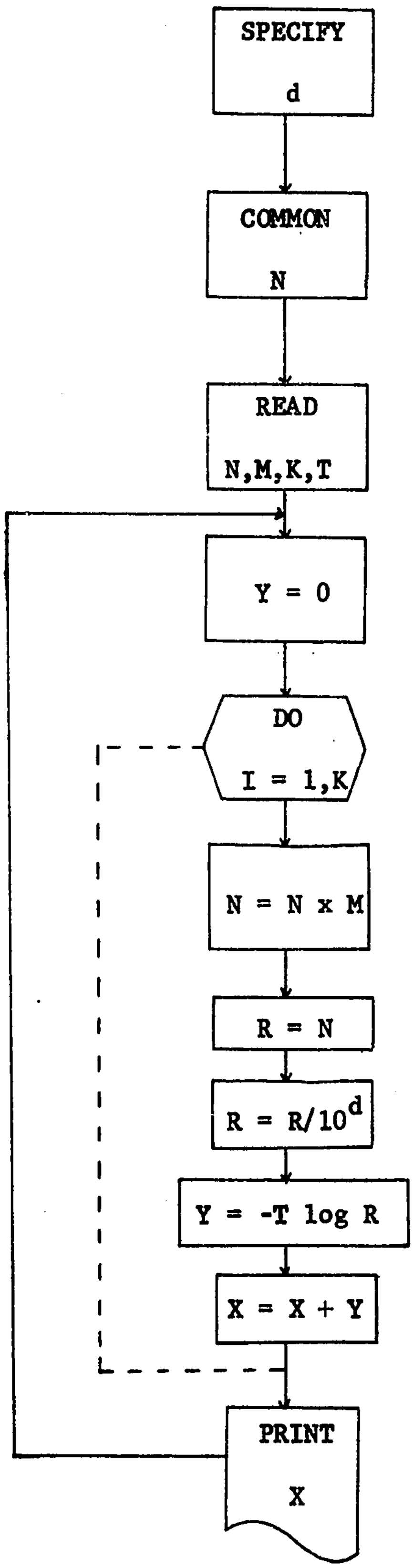
Appendix Figure 3. Generation of Exponentially Distributed Variate



T is the mean and the standard deviation of an exponential distribution.

X is the exponentially distributed variate.

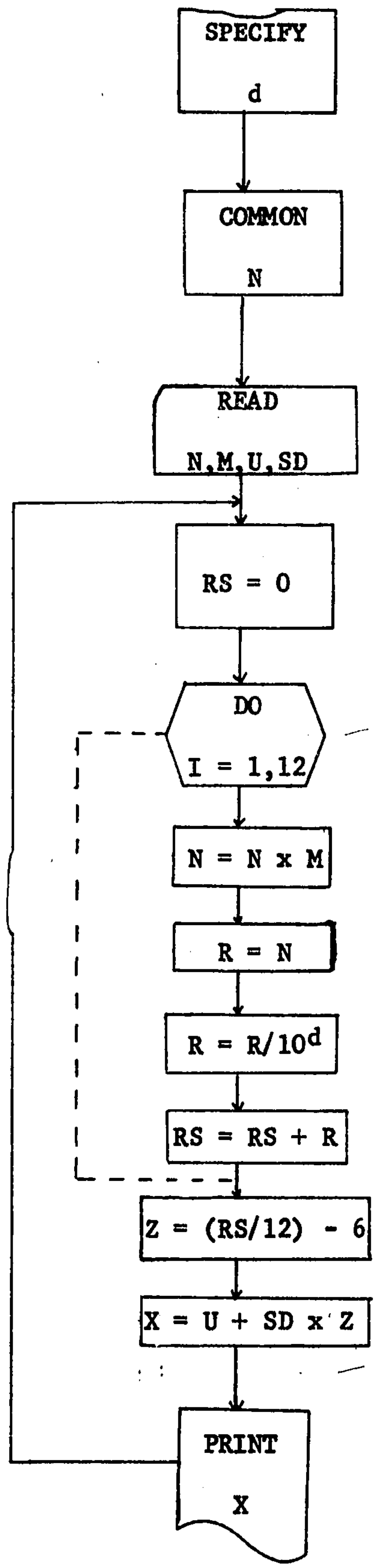
Appendix Figure 4. Generation of the Variate of a kth Erlang Gamma Distribution



T is the common mean of k exponential distributions.

X is the variate of the kth Erlang Gamma distribution.

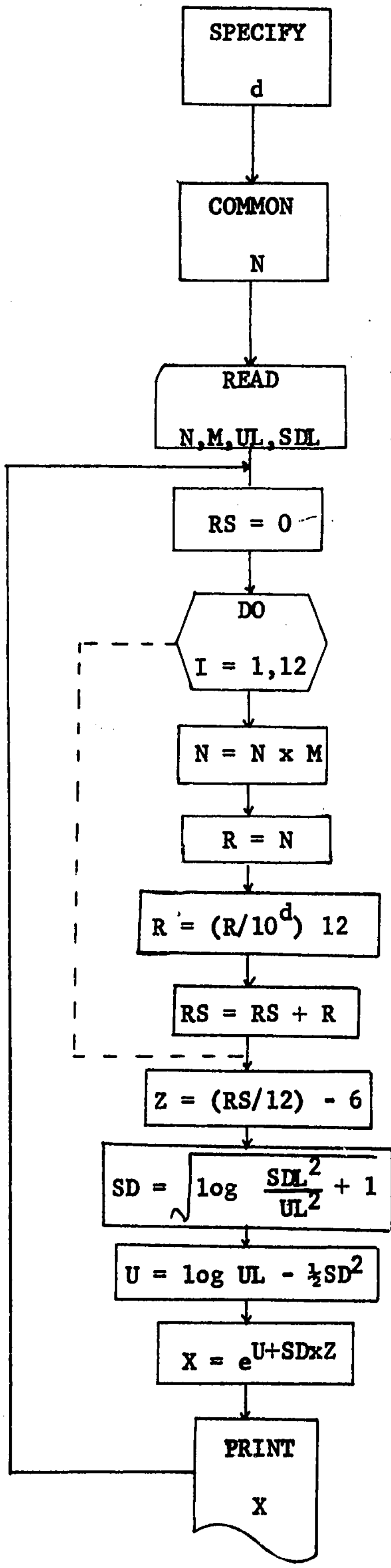
Appendix Figure 5. Generation of Normally Distributed Variate



U is the mean and SD is the standard deviation of a normal distribution.

X is the normally distributed variate.

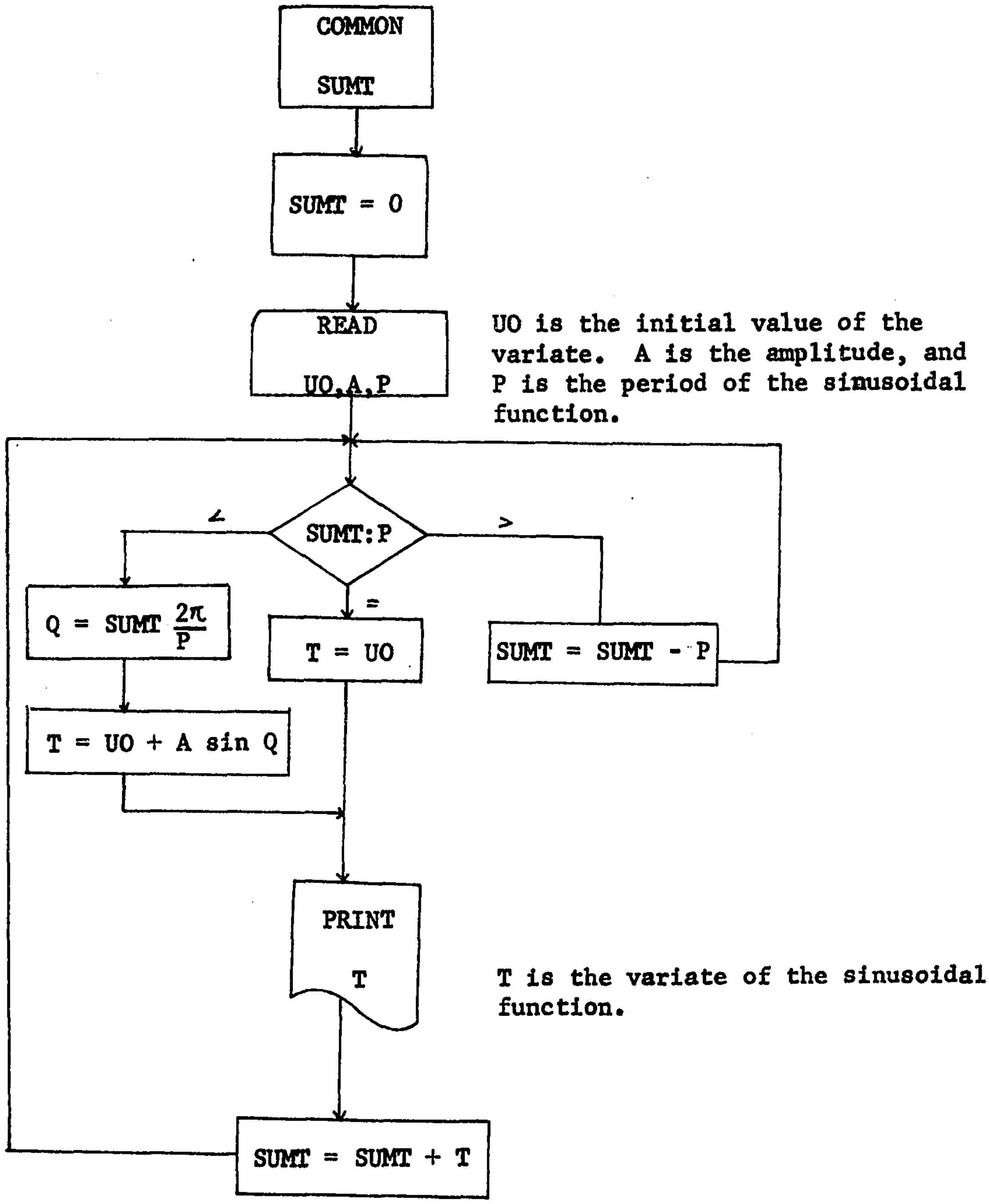
Appendix Figure 6. Generation of Lognormally Distributed Variate



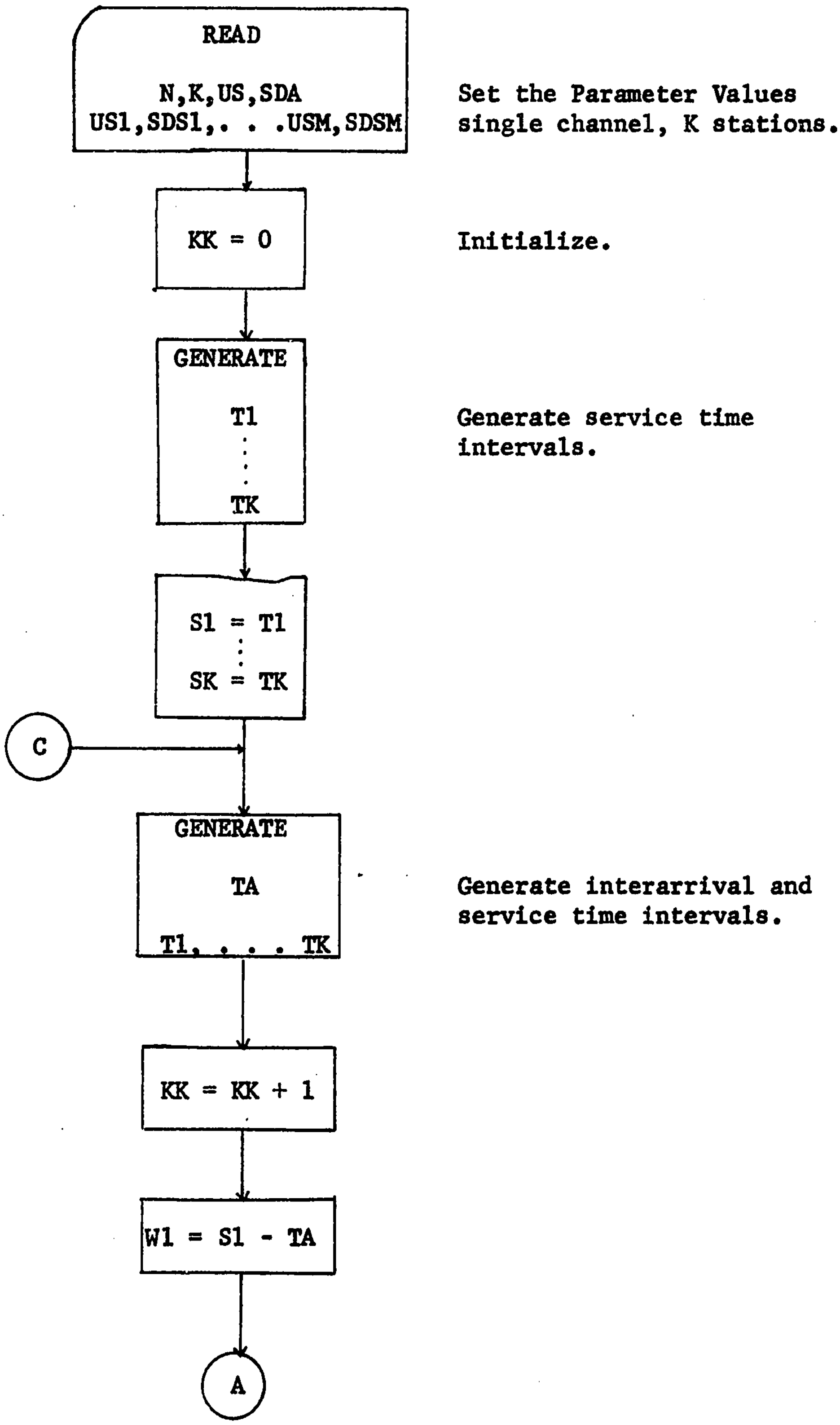
UL is the mean and SDL is the standard deviation of a lognormal distribution.

X is the lognormally distributed variate.

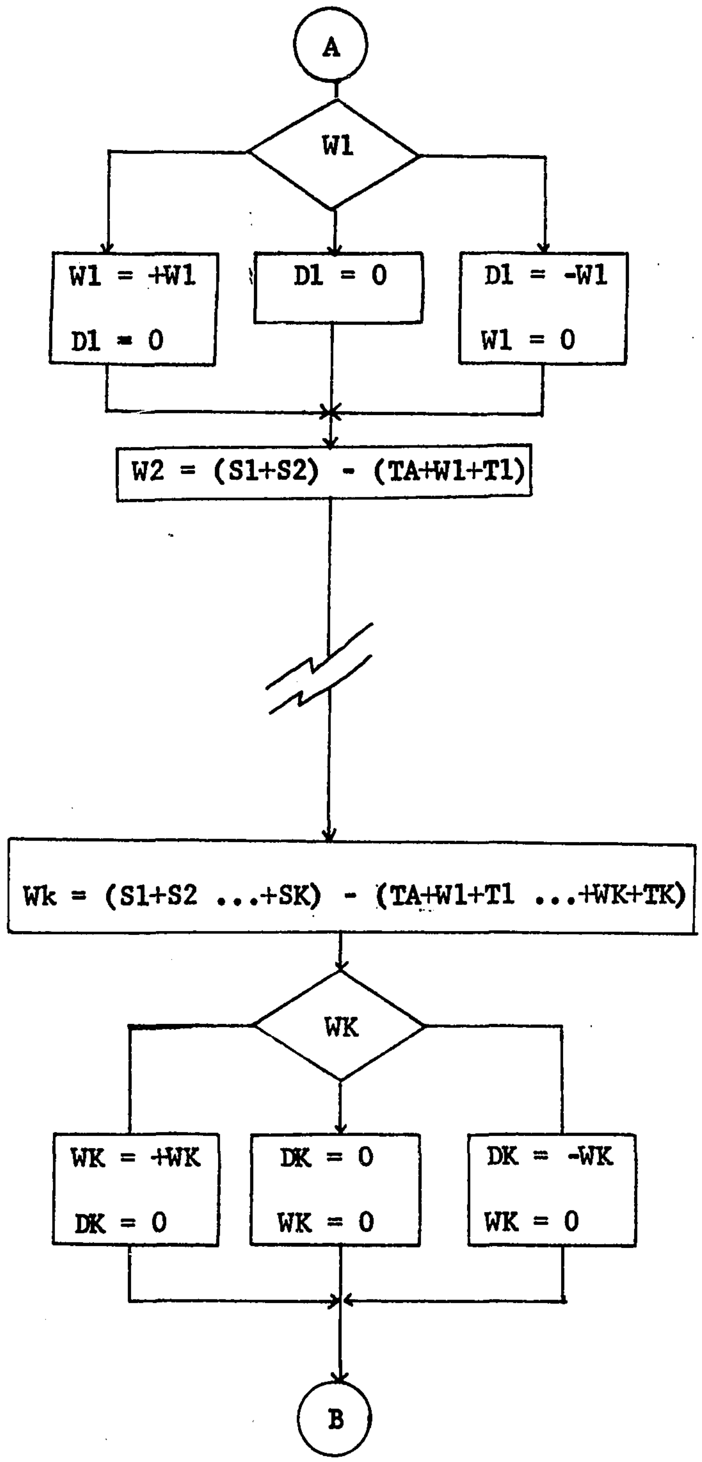
Appendix Figure 7. Generation of the Variate of a Sinusoidal Function



Appendix Figure 8a. Block Diagram of a Single Channel Waiting-Line Model



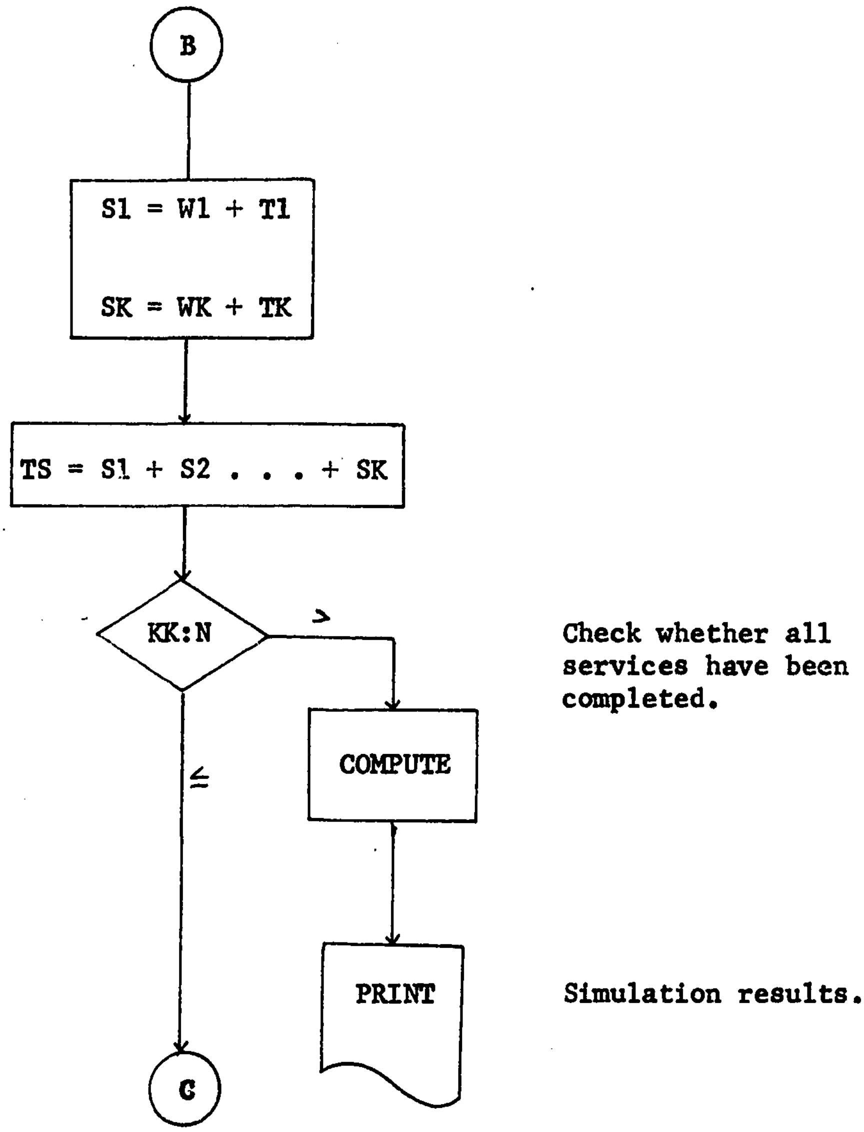
8b.



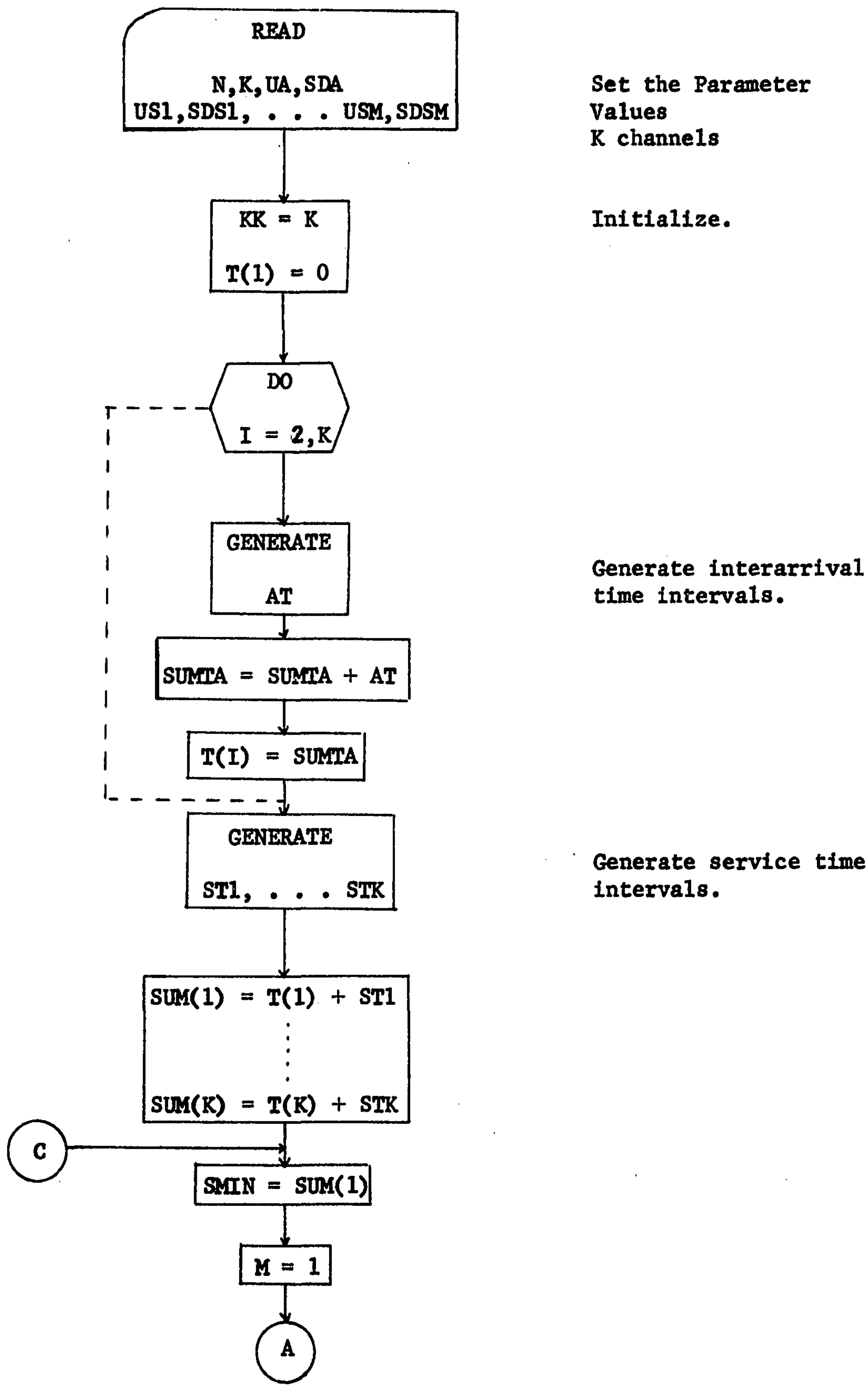
Record waiting time and idle time at station 1.

Record waiting time and idle time at station K

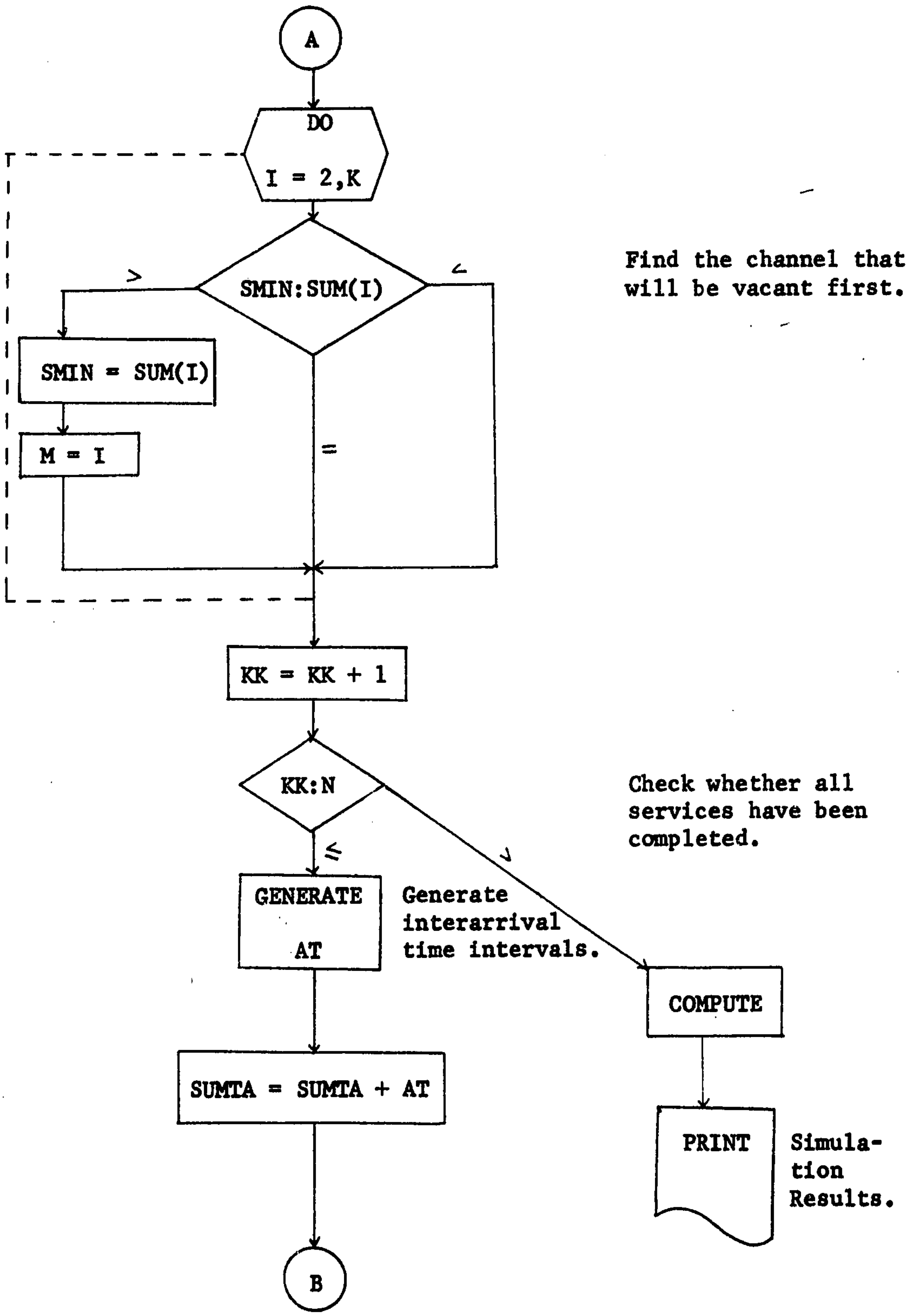
8c.


B
S1 = W1 + T1
SK = WK + TK
TS = S1 + S2 . . . + SK
KK:N
>
≤
COMPUTE
PRINT
C
Check whether all services have been completed.
Simulation results.

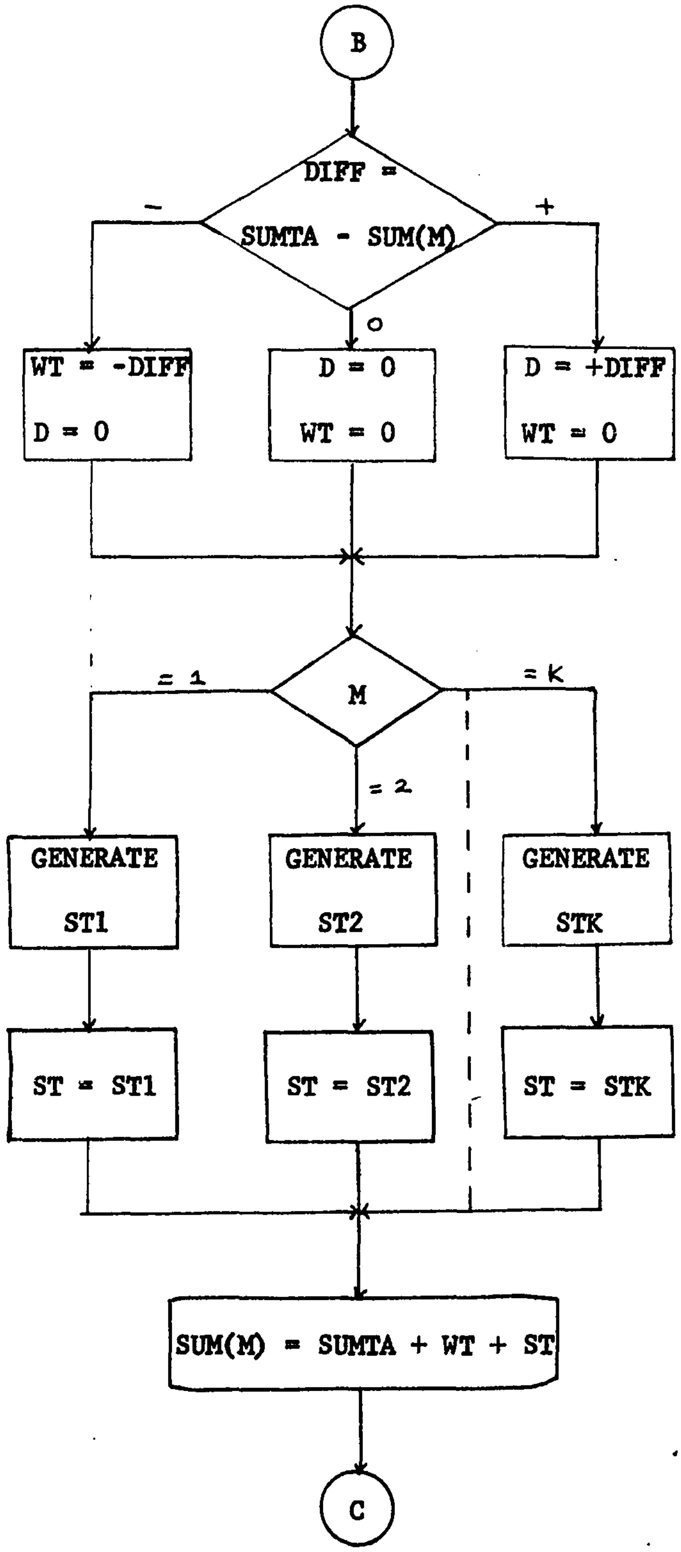
Appendix Figure 9a. Block Diagram of a Multi-Channel Waiting-Line Model



9b.


A
DO
I = 2,K
SMIN:SUM(I)
>
<
SMIN = SUM(I)
M = I
=
Find the channel that will be vacant first.
KK = KK + 1
KK:N
≤
>
Check whether all services have been completed.
GENERATE
AT
Generate interarrival time intervals.
COMPUTE
SUMTA = SUMTA + AT
PRINT
Simulation Results.
B

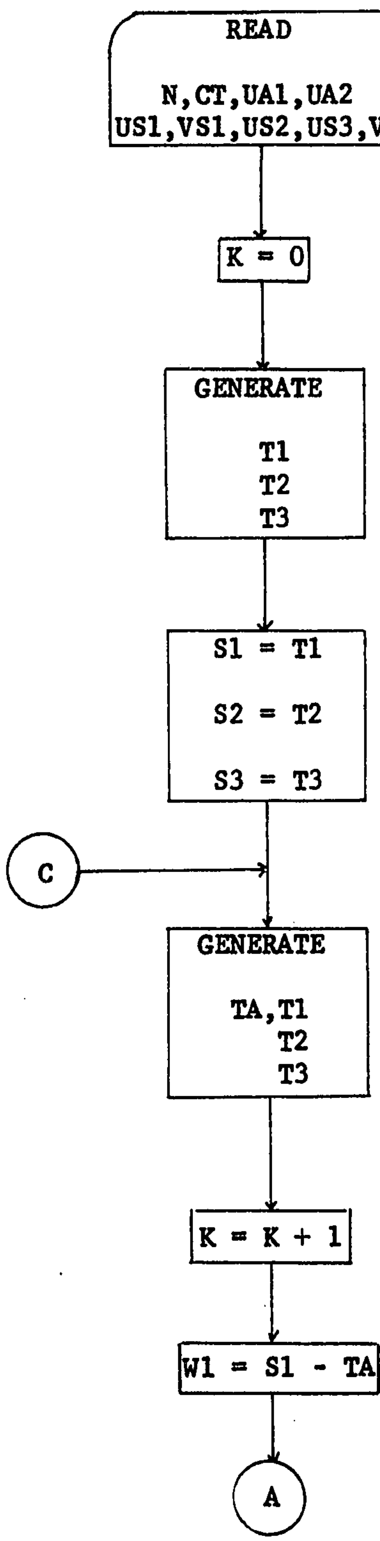
9c.



Record waiting time and idle time.

Generate service time intervals.

Appendix Figure 10a. Block Diagram of a Single Channel Three-Station Model with Output-Input Feedbacks



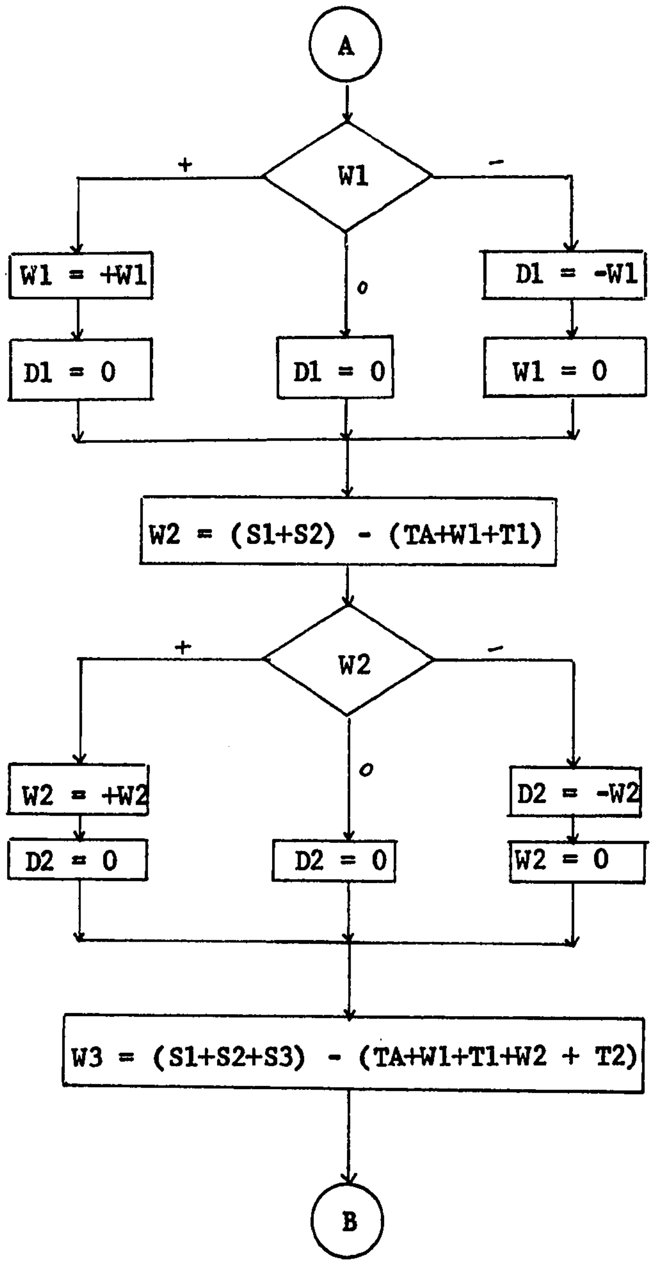
Set Parameter Values.

Initialize.

Generate service time intervals.

Generate inter-arrival and service time intervals.

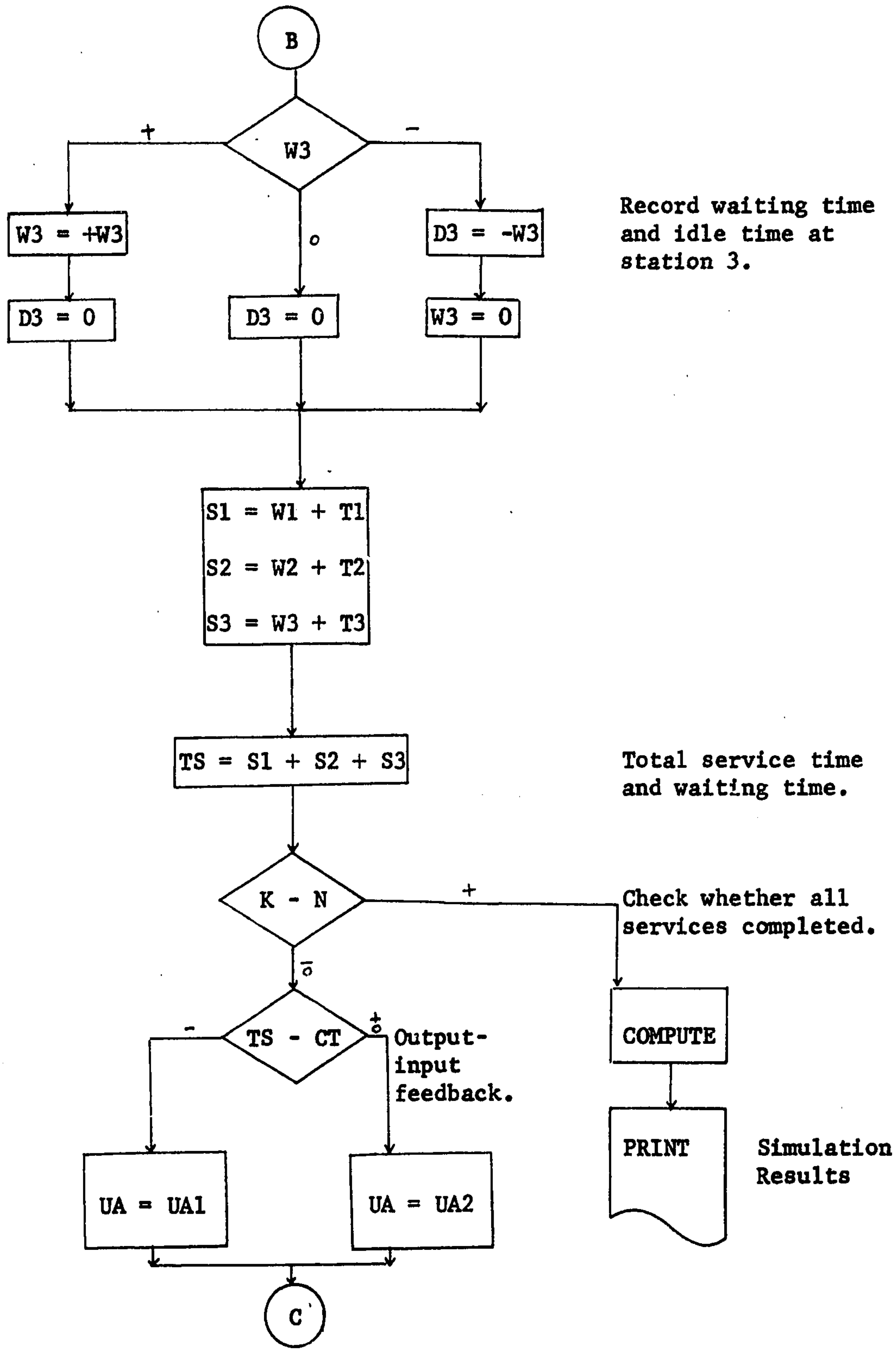
10b.



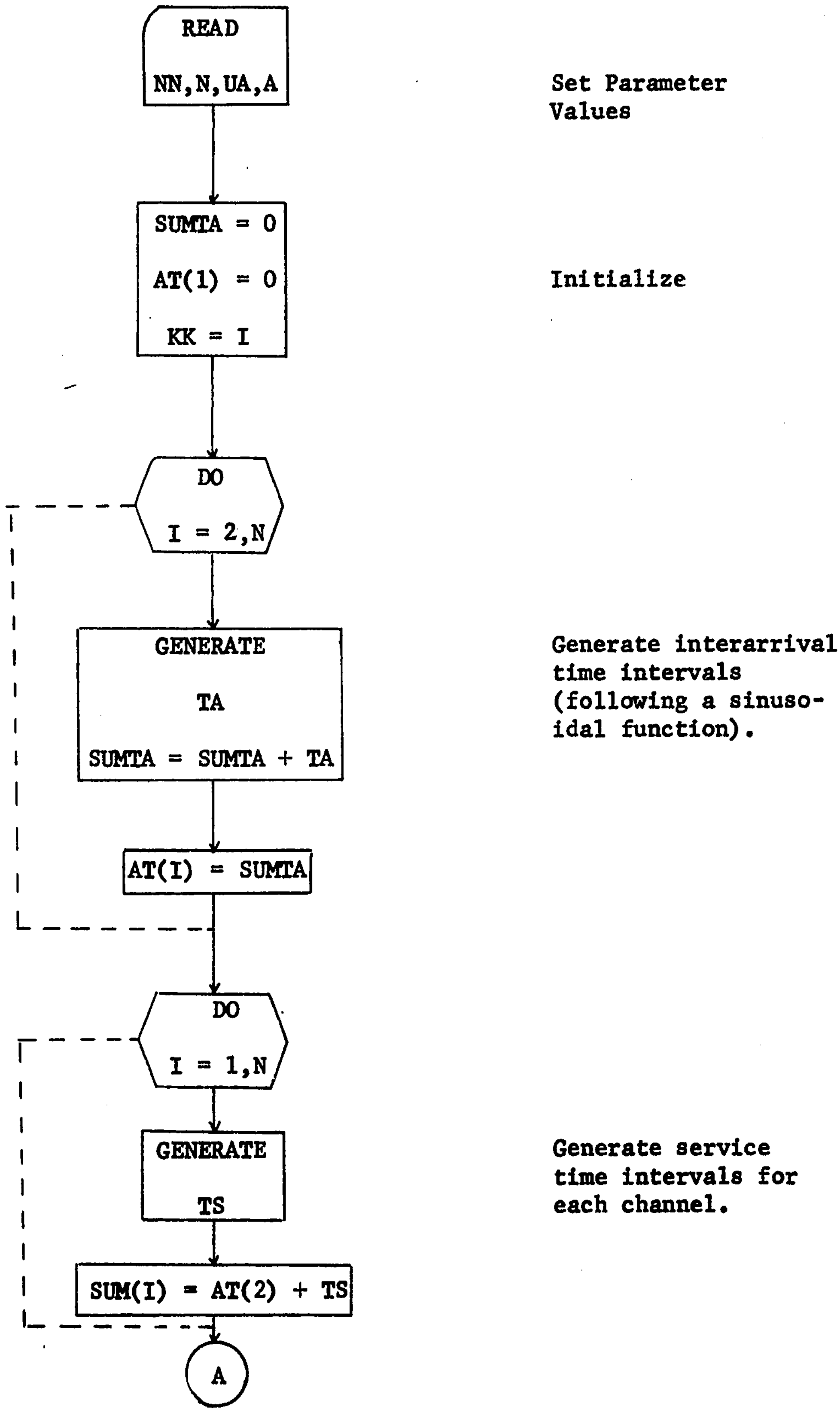
Record waiting time and idle time at station 1.

Record waiting time and idle time at station 2.

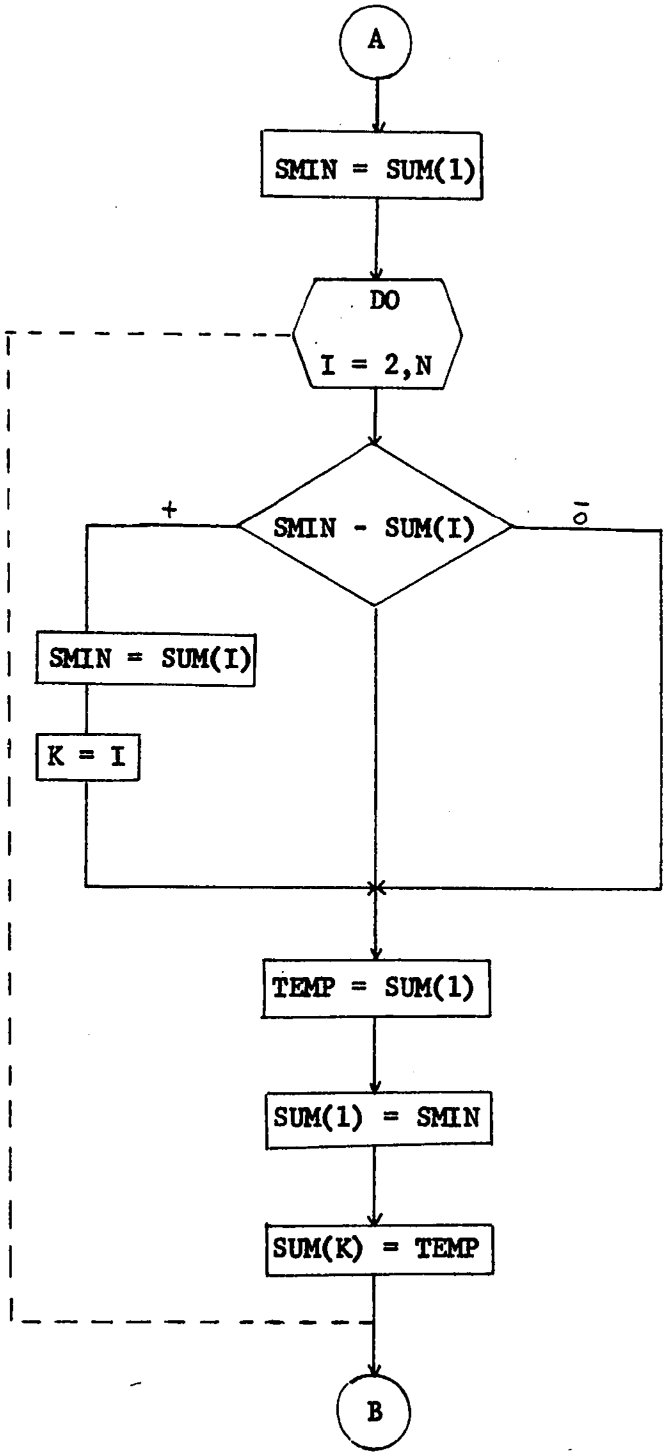
10c.


B
W3
+
-
o
W3 = +W3
D3 = -W3
D3 = 0
D3 = 0
W3 = 0
Record waiting time and idle time at station 3.
S1 = W1 + T1
S2 = W2 + T2
S3 = W3 + T3
TS = S1 + S2 + S3
Total service time and waiting time.
K - N
+
Check whether all services completed.
COMPUTE
PRINT
Simulation Results
TS - CT
Output-input feedback.
UA = UA1
UA = UA2
C

Appendix Figure 11a. Block Diagram of a Multi-Channel Model with Fluctuating Input Rate

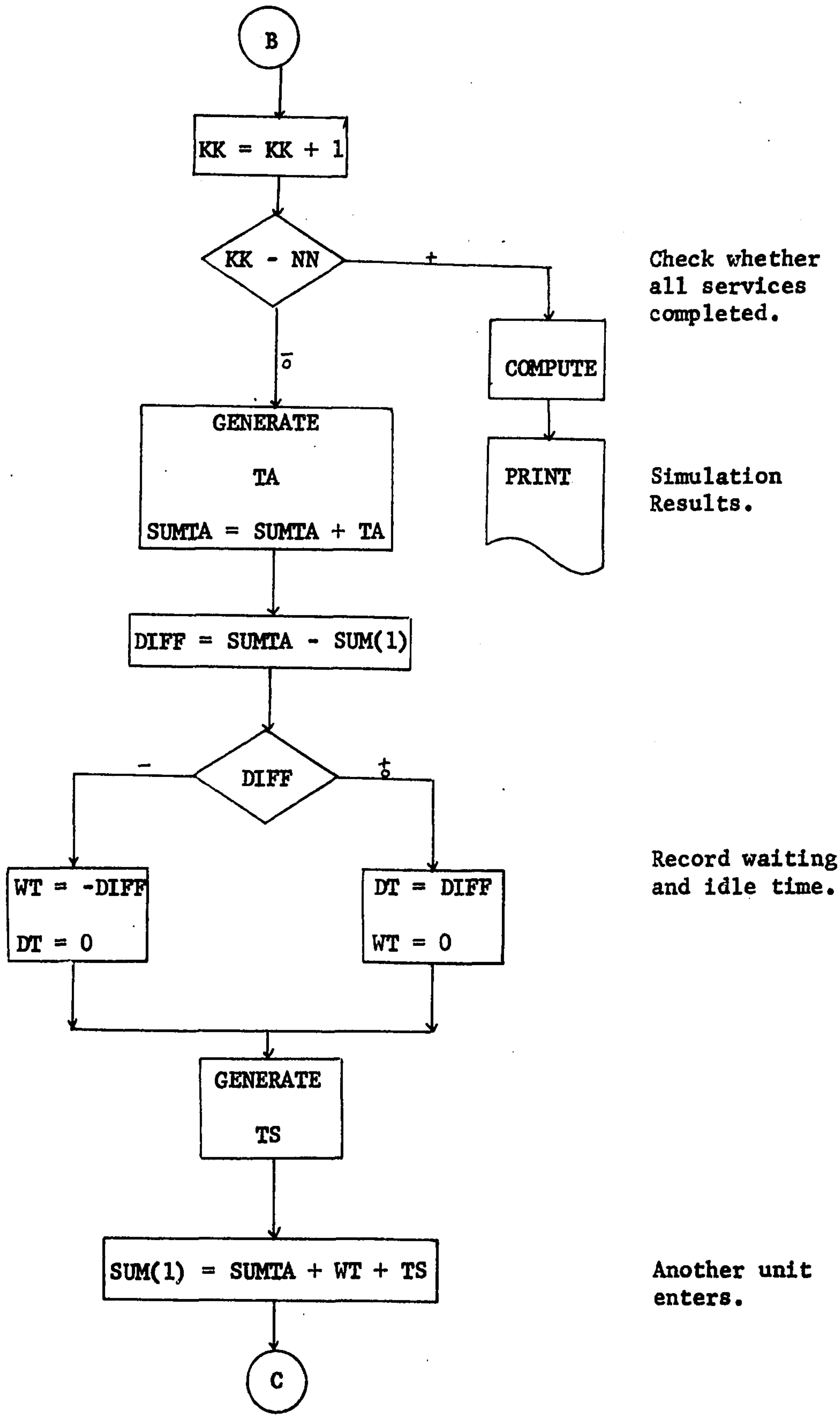


11b.

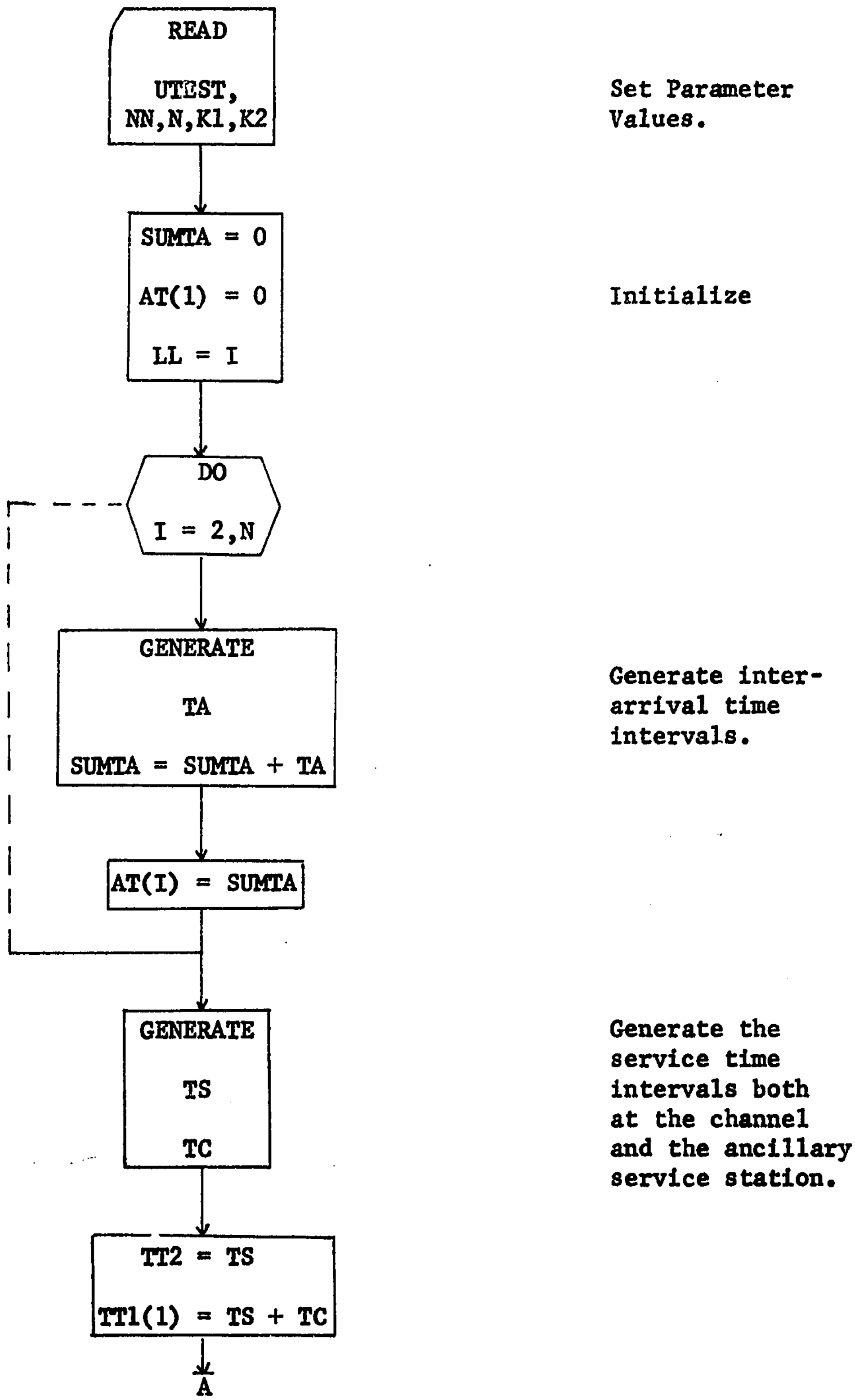


Find the channel which will be vacant first.

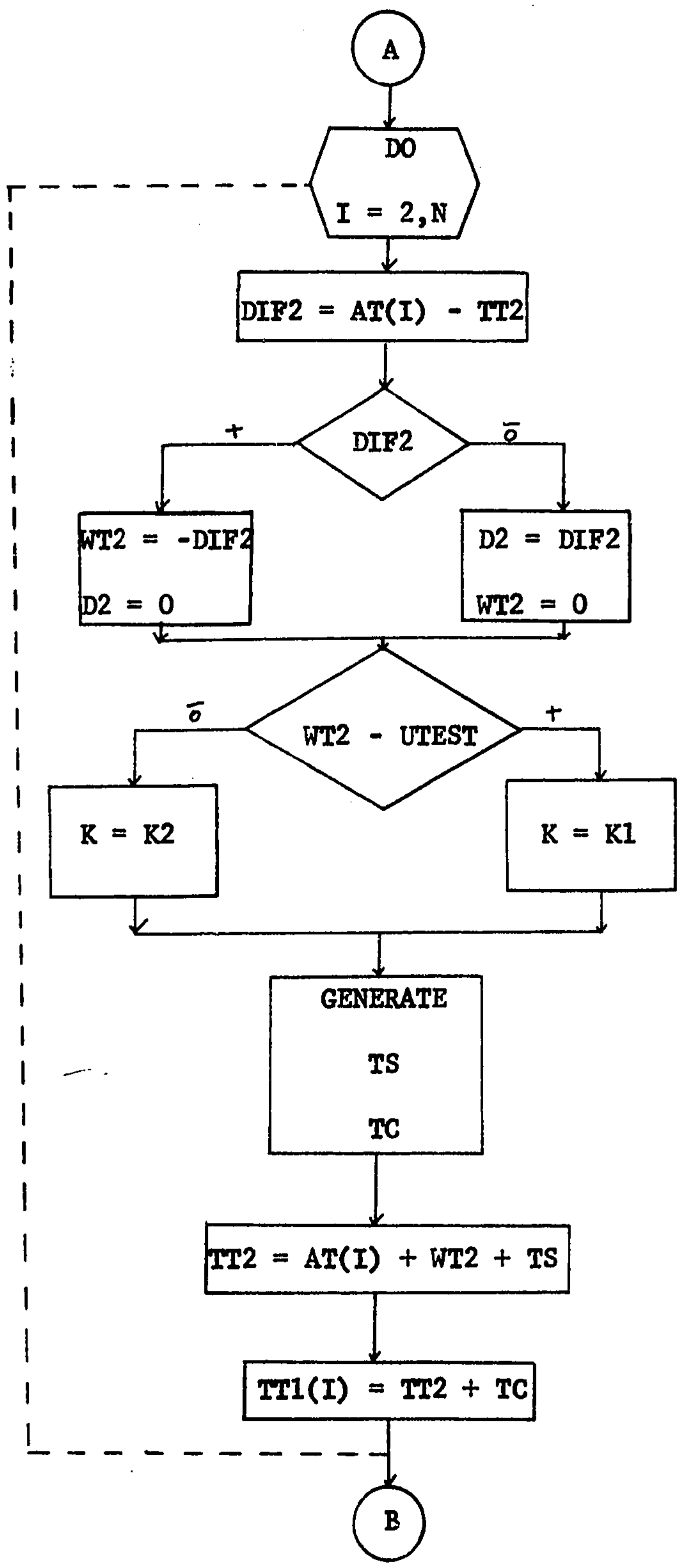
11c.


B
KK = KK + 1
KK - NN
+
0
COMPUTE
PRINT
Check whether all services completed.
GENERATE TA
SUMTA = SUMTA + TA
Simulation Results.
DIFF = SUMTA - SUM(1)
DIFF
-
+
0
WT = -DIFF
DT = 0
DT = DIFF
WT = 0
Record waiting and idle time.
GENERATE TS
SUM(1) = SUMTA + WT + TS
Another unit enters.
C

Appendix Figure 12a. Block Diagram of a Multi-Channel Model with an Attached Ancillary Service Station (Number of Services performed on each unit at the Ancillary Service Station is dependent on the Length of the Waiting Time)



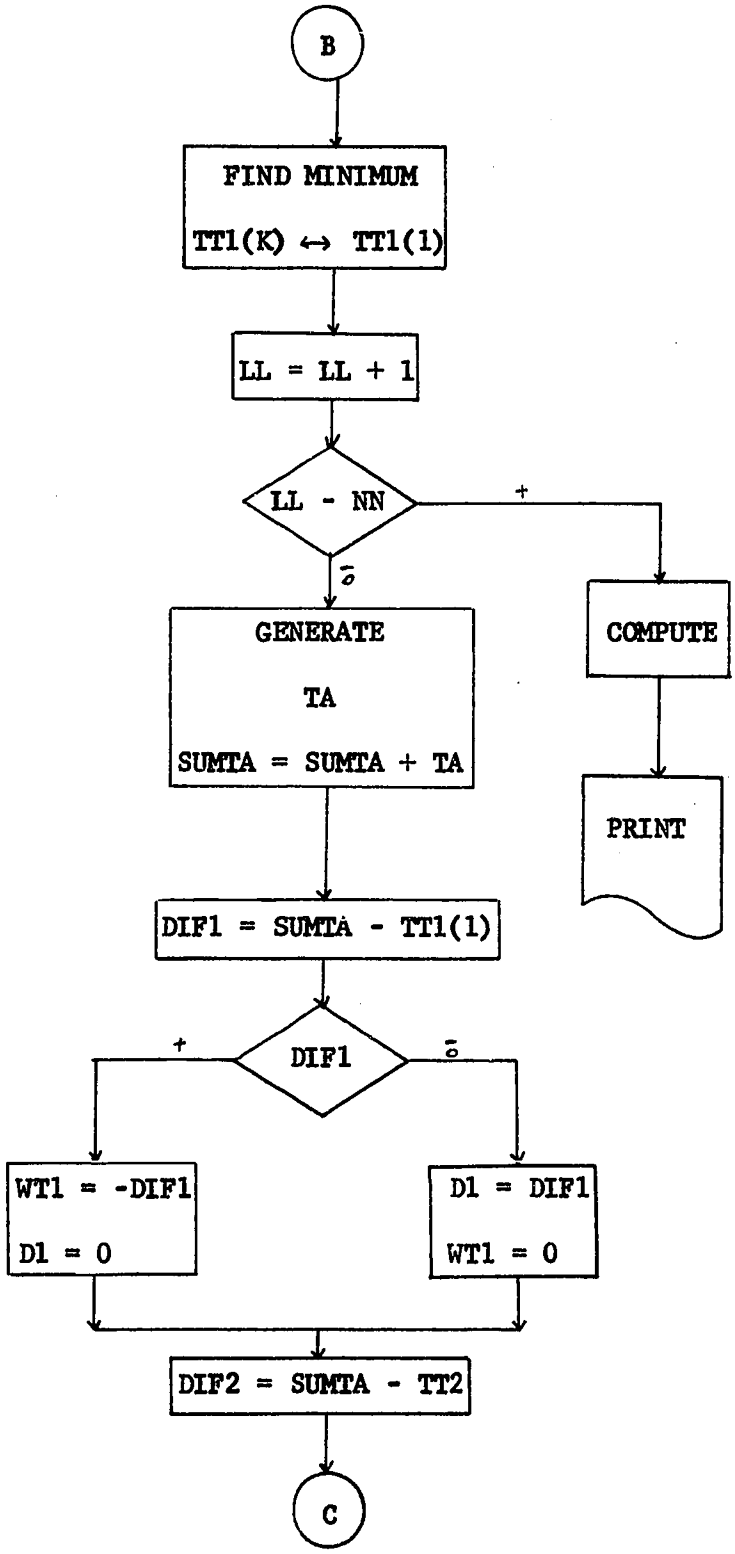
12b.



Record waiting and idle time at the ancillary service station.

Length of waiting time determines the number of services required at the ancillary service station.

12c.



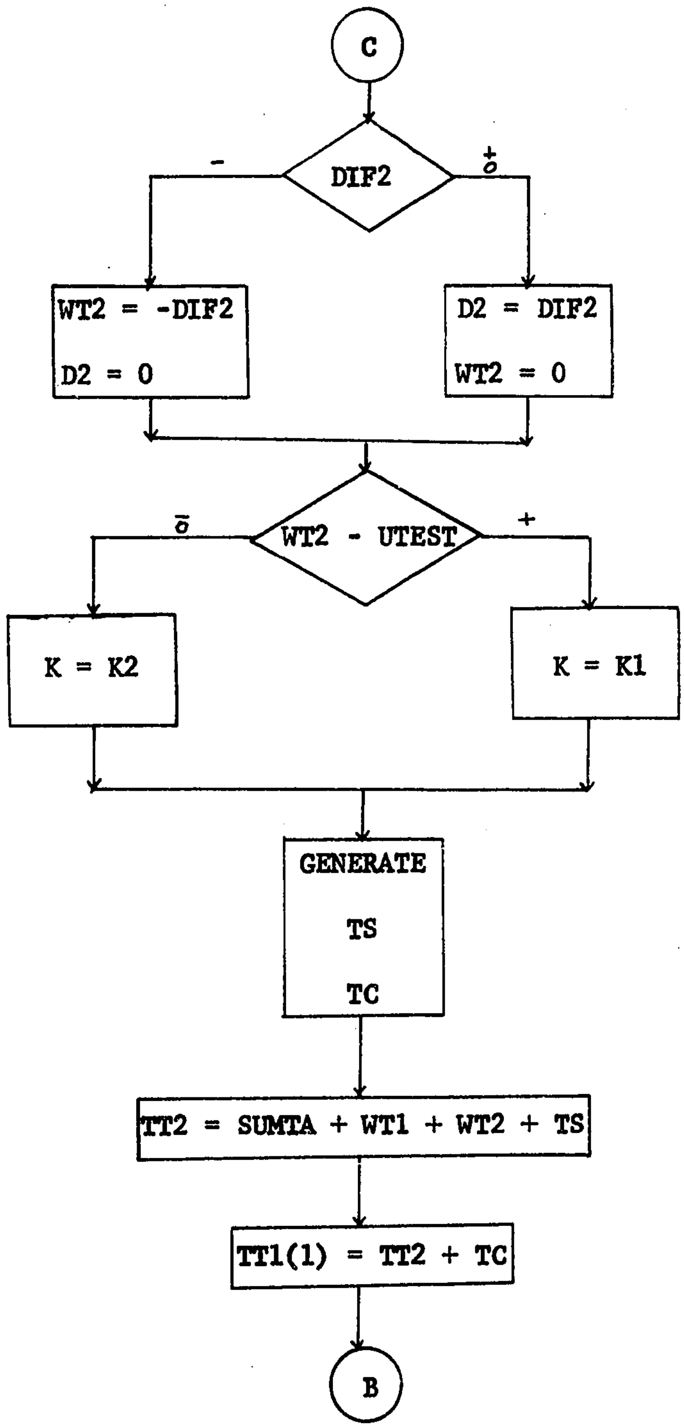
Find the channel which will be vacant first.

Check whether all services completed.

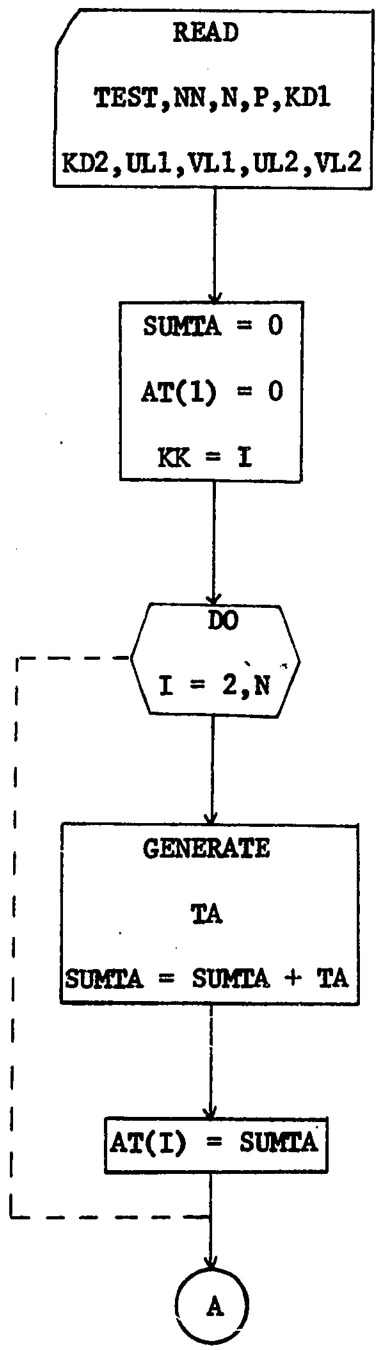
Simulation Results.

Record waiting and idle time at the channels.

12d.


C
DIF2
-
+
0
WT2 = -DIF2
D2 = 0
D2 = DIF2
WT2 = 0
WT2 - UTEST
0
-
+
K = K2
K = K1
GENERATE
TS
TC
TT2 = SUMTA + WT1 + WT2 + TS
TT1(1) = TT2 + TC
B

Appendix Figure 13a. Block Diagram of a Multi-Channel Model with Two Intervals of Service Time (the Length of the Second Interval is dependent on the length of the First)

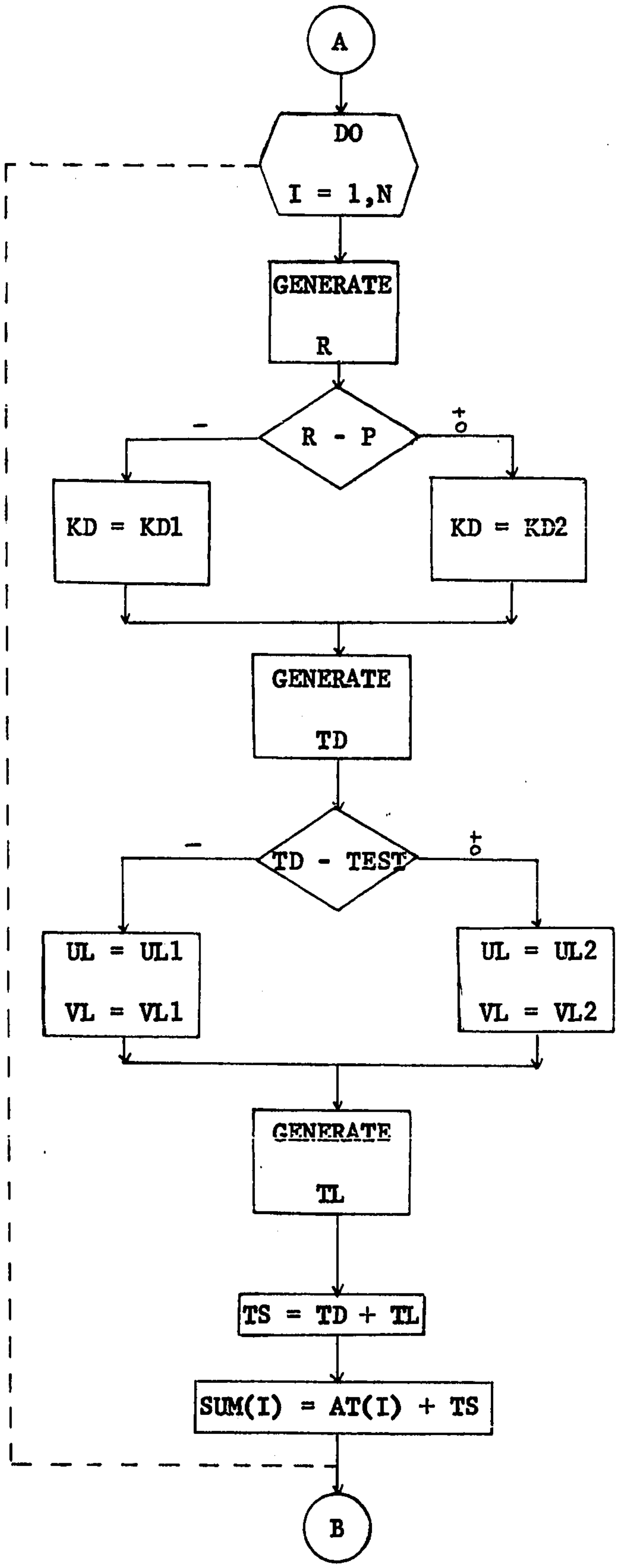


Set Parameter Values.

Initialize

Generate inter-arrival time intervals.

13b.



Generate random numbers.

Determine the parameter of the distribution of the first portion of the service time interval.

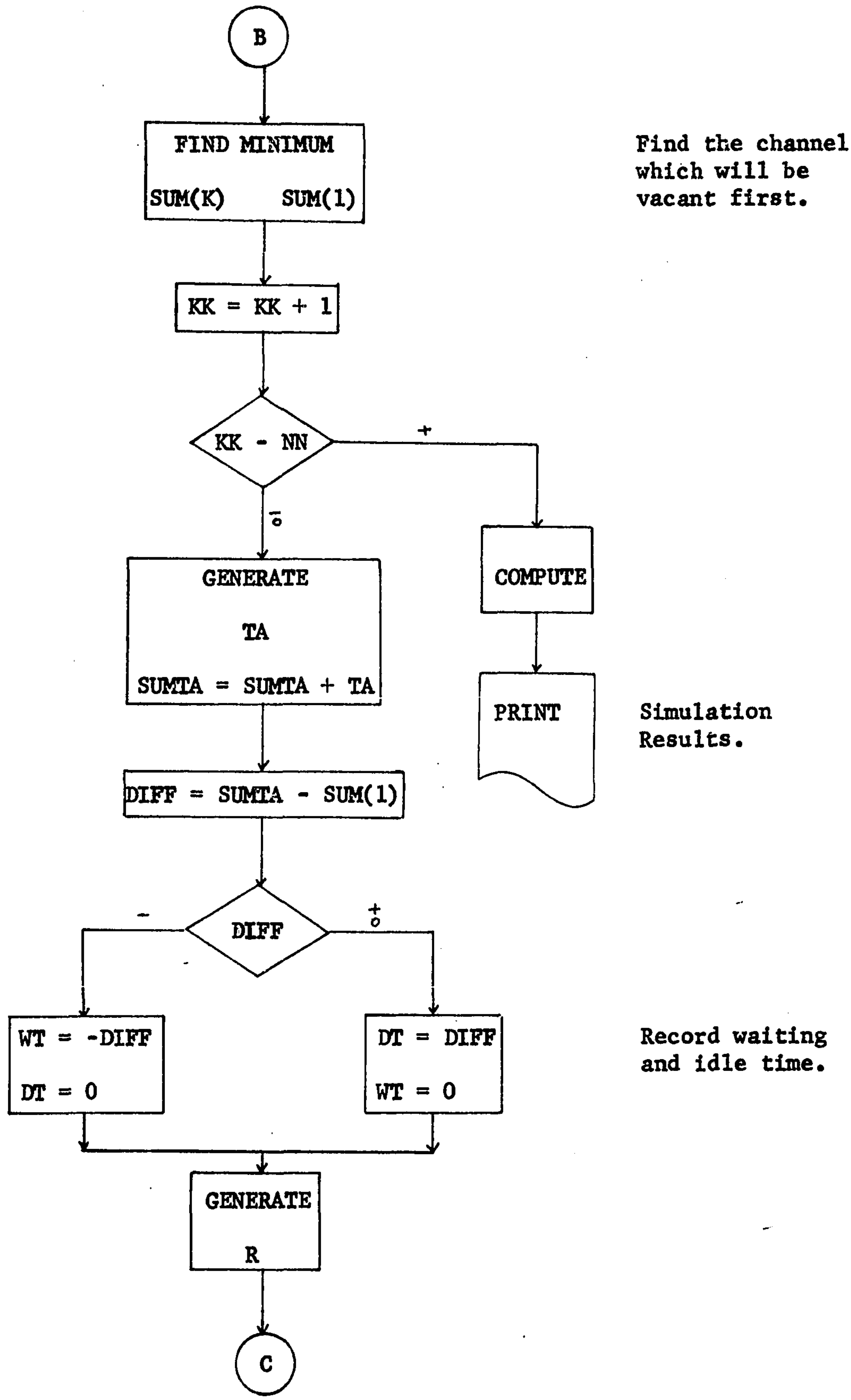
Generate the first portion of the service time interval.

Determine the parameters of the distribution of the second portion of the service time interval.

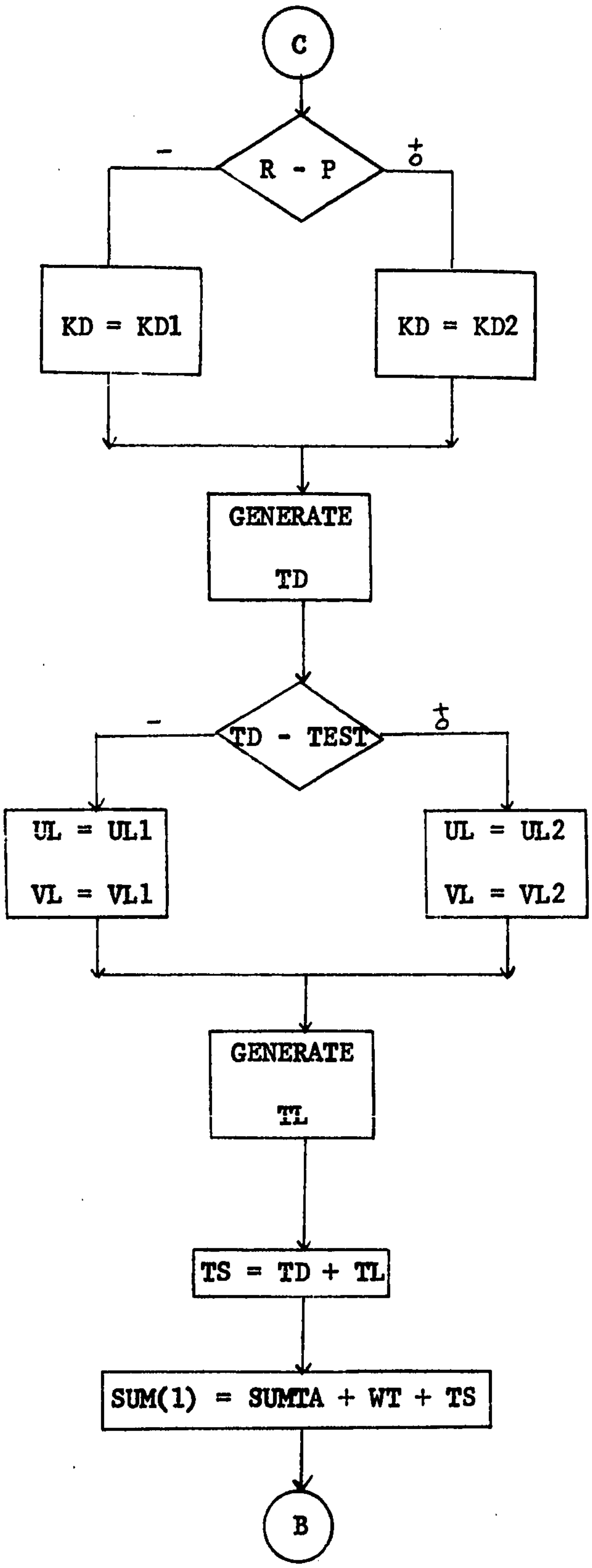
Generate the second portion of the service time interval.

Total service time interval.

13c.


B
FIND MINIMUM
SUM(K) SUM(1)
Find the channel which will be vacant first.
KK = KK + 1
KK - NN
+
-0
COMPUTE
PRINT
Simulation Results.
GENERATE
TA
SUMTA = SUMTA + TA
DIFF = SUMTA - SUM(1)
DIFF
-
+0
WT = -DIFF
DT = 0
DT = DIFF
WT = 0
Record waiting and idle time.
GENERATE
R
C

13d.


C
R - P
-
+
0
KD = KD1
KD = KD2
GENERATE
TD
TD - TEST
-
+
0
UL = UL1
VL = VL1
UL = UL2
VL = VL2
GENERATE
TL
TS = TD + TL
SUM(1) = SUMTA + WT + TS
B

APPENDIX B

FORTRAN PROGRAMS

Appendix B contains the FORTRAN subprograms for the generation of the probability distributions and the FORTRAN main programs of the six models used in this investigation.

FORTRAN Subprogram 1. Random Number Generation

```
SUBROUTINE RANDOM (R)
COMMON N,M
N=N*M
R=N
R=R/1.E8
RETURN
END
```

FORTRAN Subprogram 2. Exponential Variate Generation

```
SUBROUTINE EXPNT(T,U)
COMMON N,M
N=N*M
R=N
R=R/1.E8
T=-U*LOGF(R)
RETURN
END
```

FORTRAN Subprogram 3. Erlang Gamma Variate Generation

```
 1 SUBROUTINE GAMMAK(T,K,U)
 2 COMMON N,M
 3 T=0.
 4 N=N*M
 5 DO 9 I=1,K
 6 R=N
 7 R=R/1.E8
 8 ST=-U*LOGF(R)
 9 T=T+ST
10 RETURN
11 END
```

FORTRAN Subprogram 4. Normal Variate Generation

```
1 SUBROUTINE NORM(X,U,DS)
2 COMMON N,M
3 RS=0.
4 I=0
5 DO 9 I=1,12
6 N=N*M
7 R=N
8 R=R/1.E8
9 RS=RS+R
10 Z=(RS/12.)-6.
11 X=U+SD*Z
12 RETURN
13 END
```

FORTRAN Subprogram 5. Lognormal Variate Generation

```
1 SUBROUTINE LGNORM(T,UL,SDL)
2 COMMON N,M
3 RS=0.
4 DO 8 I=1,12
5 N=N*M
6 R=N
7 R=R/1.E8
8 RS=RS+R
9 Z=(RS/12.)-6.
10 W=LOGF((SDL/UL)**2+1.)
11 V=W**0.5
12 U=LOGF(UL)-W/2.
13 T=EXPF(U+V*Z)
14 RETURN
15 END
```

FORTRAN Subprogram 6. Sinusoidal Variate Generation

```
1 SUBROUTINE SINUO(T,UO,A,P)
2 COMMON N,M,SUMT
3 IF (SUMT-P) 8,6,4
4 SUMT=SUMT-P
5 GO TO 3
6 T=UO
7 GO TO 10
8 Q=(SUMT*6.2832)/P
9 T=UO+A*SINF(Q)
10 SUMT=SUMT+T
11 RETURN
12 END
```

FORTRAN Main Program 1. Single Channel Model

```
      COMMON NO,MO,N1
    1 FORMAT (3I8)
  100 READ 1,NO,MO,N1
    2 FORMAT (I4)
      READ 2,NT
      SN=NT
    3 FORMAT (2F5.0)
      READ 3,UA,U1
      K=1
      SUMAT=0.
      SUMQA-0.
      W1=0.
      SQW=0.
      D1=0.
      SQD=0.
      SS1=0.
      SW1=0.
      SD1=0.
      CALL EXPNS (T1,U1)
      S1=T1
      SUMT1=T1
      SUMQ1=T1*T1
    4 CALL EXPNT (TA,UA)
      CALL EXPNS(T1,U1)
      K=K+1
      IF (K-NT) 5,5,10
    5 SUMAT=SUMAT+TA
```

```
      SUMQA=SUMQA+TA*TA
      SUMT1=SUMT1+T1
      SUMQ1=SUMQ1+T1*T1
      W1=S1-TA
      IF (W1) 6,7,7
    6 SD1=SD1-W1
      D1=-W1
      SQD=SQD+D1*D1
      W1=0.
      GO TO 8
    7 SW1=SW1+W1
      D1=0.
      SQW=SQW+W1*W1
    8 S1=W1+T1
      SS1=SS1+S1
      GO TO 4
   10 AA=SUMAT/(SN-1.)
      AS1=SUMT1/SN
      DL=SUMT1/(SD1+SUMT1)
      AWT1=SW1/SN
      AD=SD1/SN
      SDD=((SQD-SD1**2/SN)/(SN-1.))**.5
      SDW=((SQW-SW1**2/SN)/(SN-1.))**.5
      SDTA=((SUMQA-SUMAT**2/(SN-1.))/(SN-2.))**.5
      SDT1=((SUMQ1-SUMT1**@/SN)/(SN-1.))**.5
   11 FORMAT (1HK,4F20.2)
      PRINT 11,AA,AS1,AWT1,AD,SDTA,SDT1,SDW,SDD,SD1,SW1,SS1,DL
      GO TO 100
      END
```

FORTRAN Main Program 2. Multi-Channels Model

```
  1 DIMENSION AT(10),SUM(10),NA(10)
    COMMON NO,MO,NA
500 FORMAT (10I3)
    READ 500,NO,MO,NA
  2 FORMAT (4F5.2)
  3 READ 2,US,UA
  4 FORMAT (I3,I4)
  5 READ 4,N,NN
  6 S=N
  7 SN=NN
    SNS=SN-1.
  8 SUMTA=0.
  9 SUMTS=0.
 10 SQTA=0.
 11 SQTS=0.
 12 TWT=0.
 13 TST=0.
    TWQT=0.
    SQW=0.
    TDT=0.
    SQD=0.
 14 KK=N
 20 AT(1)=0.
 21 DO 26 I=2,N
 22 CALL EXPNT(TA,UA)
 24 SUMTA=SUMTA+TA
    SQTA=SQTA+TA*TA
```

```
26 AT(I)=SUMTA
27 DO 31 I=1,N
28 CALL EXPNS(TS,US,I)
   SUMTS=SUMTS+TS
   SQTS=SQTS+TS*TS
31 SUM(I)=AT(I)+TS
34 SMIN=SUM(1)
   K=1
35 DO 40 I=2,N
37 IF (SMIN-SUM(I)) 40,40,38
38 SMIN=SUM(I)
39 K=I
40 CONTINUE
41 TEMP=SUM(1)
42 SUM(1)=SMIN
43 SUM(K)=TEMP
45 KK=KK+1
46 IF (KK-NN) 47,47,100
47 CALL EXPNT(TA,UA)
48 SUMTA=SUMTA+TA
49 SQTA=SQTA+TA*TA
50 DIFF=SUMTA-SUM(1)
51 IF (DIFF) 52,55,55
52 WT=-DOFF
   D=0.
53 TWT=TWT+WT
   SQW=SQW+WT*WT
54 GO TO 57
55 WT=0.
```

```
    D=DIFF
    TDT=TDT+D
    SQD=SQD+D*D
 57 CALL EXPNS(TS,US,K)
 59 SUMTS=SUMTS+TS
 60 SQTS=SQTS+TS*TS
 61 SUM(1)=SUMTA+WT+TS
 66 GO TO 34
100 AA=SUMTA/SNS
    DL=SUMTS/(SUMTS+TDT)
101 CALL VARSD(SDA,SUMTA,SQTA,SNS)
102 AS=SUMTS/SN
103 CALL VARSD(SDS,SUMTS,SQTS,SN)
104 AW=TWT/SN
    CALL VARSD(SDW,TWT,SQW,SN)
    AD=TOT/SN
    CALL VARSD(SDD,TDT,SQD,SN)
106 FORMAT (1HK,5X,I4,5X,I3//(1HK,2F20.2))
107 PRINT 106,NN,N,AA,SDA,AS,SDS,AW,AD,SDD,DL
109 GO TO 500
110 END
```

FORTRAN Main Program 3. Single Channel, Three Stations, Output-Input Feedbacks

```
300 FORMAT (1H1)
301 PRINT 300
  1 COMMON NO,MO,N1,N2,N3
  2 FORMAT (5I8)
  3 READ 2,NO,MO,N1,N2,N3
  4 FORMAT (2I4)
  5 READ 4,NT,L
  6 SN=NT-L
  7 FORMAT (8F5.2)
  8 READ 7,UA1,UA2,CT,U1,V1,U2,U3,V3
200 UA=UA1
  9 K=1
 10 M=0
 11 SUMAT=0.
 12 SS1=0.
 13 SS2=0.
 14 SS3=0.
 18 SD1=0.
 19 SD2=0.
 20 SD3=0.
 21 W1=0.
 22 W2=0.
 23 W3=0.
 24 SW1=0.
 25 SW2=0.
 26 SW3=0.
```

```
      SQD1=0.
      SQD2=0.
      SQD3=0.
      SQW1=0.
      SQW2=0.
      SQW3=0.
   27 TA=0.
      SUMT1=0.
      SUMT2=0.
      SUMT3=0.
      SUMQA=0.
      SUMQ1=0.
      SUMQ2=0.
      SUMQ3=0.
      KK=0
  100 CALL NORM(T1,U1,V1)
      IF (T1) 100,100,101
  101 CALL EXPNS(T2,U2)
  102 CALL LGNORM(T3,U3,V3)
  103 S1=T1
  104 S2=T2
  105 S3=T3
   29 CALL EXPNT(TA,UA)
   30 CALL NORM(T1,U1,V1)
      IF (T1) 30,30,31
   31 CALL EXPNS(T2,U2)
   32 CALL LGNORM(T3,U3,V3)
   33 K=K+1
```

```
34 IF (K-L) 38,36,35
35 IF (K-NT) 37,66,81
36 M=1
37 SUMAT=SUMAT+TA
   SUMQA=SUMQA+TA*TA
   SUMT1=SUMT1+T1
   SUMQ1=SUMQ1+T1*T1
   SUMT2=SUMT2+T2
   SUMQ2=SUMQ2+T2*T2
   SUMT3=SUMT3+T3
   SUMQ3=SUMQ3+T3*T3
38 W1=S1-TA
39 IF (W1) 40,43,43
40 IF (M) 80,42,41
41 SD1=SD1-W1
   D1=-W1
   SQD1=SQD1+D1*D1
42 W1=0.
   GO TO 45
43 IF (M) 80,45,44
44 SW1+SW1+W1
   SQW1=SQW1+W1*W1
   D1=0.
45 W2=(S1+S2)-(TA+T1+W1)
46 IF (W2) 47,50,50
47 IF (M) 80,49,48
48 SD2=SD2-W2
   D2=-W2
```

```
      SQD2=SQD2+D2*D2
   49 W2=0.
      GO TO 52
   50 IF (M) 80,52,51
   51 SW2=SW2+W2
      SQW2=SQW2+W2*W2
      D2=0.
   52 W3=(S1+S2+S3)-(TA+T1+T2+W1+W2)
   53 IF (W3) 54,57,57
   54 IF (M) 80,56,55
   55 SD3=SD3-W3
      D3=-W3
      SQD3=SQD3+D3*D3
   56 W3=0.
      GO TO 59
   57 IF (M) 80,59,58
   58 SW3=SW3+W3
      SQW3=SQW3+W3*W3
      D3=0.
   59 S1=W1+T1
   60 S2=W2+T2
   61 S3=W3+T3
  302 IF(M) 80,201,400
  400 KK=KK+1
  303 FORMAT (1HK,I6,5F15.2/1HK,6X,5F15.2)
  304 PRINT 303,KK,TA,W1,D1,T1,W2,D2,T2,W3,D3,T3
   62 SS1=SS1+S1
   63 SS2=SS2+S2
```

```
 64 SS3=SS3+S3
201 TS=S1+S2+S3
202 IF (TS-CT) 205,203,203
203 UA=UA2
204 GO TO 29
205 UA=UA1
 65 GO TO 29
 66 AA=SUMAT/SN
    AD1=SD1/SN
    AD2=SD2/SN
    AD3=SD3/SN
    DL1=SUMT1/(SUMT1+SD1)
    DL2=SUMT2/(SUMT2+SD2)
    DL3=SUMT3/(SUMT3+SD3)
 70 AS1=(SS1-SW1)/SN
 71 AS2=(SS2-SW2)/SN
 72 AS3=(SS3-SW3)/SN
    AS=AS1+AS2+AS3
 73 AWT1=SW1/SN
 74 AWT2=SW2/SN
 75 AWT3=SW3/SN
    SDTA=((SUMQA-SUMAT**2/SN)/(SN-1.))**.5
    SDT1=((SUMQ1-SUMT1**2/SN)/(SN-1.))**.5
    SDT2=((SUMQ2-SUMT2**2/SN)/(SN-1.))**.5
    SDT3=((SUMQ3-SUMT3**2/SN)/(SN-1.))**.5
    SDW1=((SQW1-SW1**2/SN)/(SN-1.))**.5
    SDW2=((SQW2-SW2**2/SN)/(SN-1.))**.5
    SDW3=((SQW3-SW3**2/SN)/(SN-1.))**.5
```

```
      SDD1=((SQD1-SD1**2/SN).(SN-1.))**.5
      SDD2=((SQD2=SD2**2/SN)/(SN-1.))**.5
      SDD3=((SQD3-SD3**2/SN)/(SN-1.))**.5
   76 FORMAT (1HK,7F15.2/1HK,7F15.2/(1HK,3F15.2))
   77 PRINT 76,AA,AWT1,AS1,AWT2,AS2,AWT3,AS3,SDTA,SDW1,
     1SDW2,SDT2,SDW3,SDT3,AD1,AD2,AD3,SDD1,SDD2,SDD3,
     2DL1,DL2,DL3
   79 GO TO 5
   80 STOP 1
   81 STOP 2
   82 END
```

FORTRAN Main Program 4. Multi-Channels, Fluctuating Input Rate

```
  1 DIMENSION AT(100),SUM(100),NA(100)
    COMMON NO,MO,SMT,NA
400 FORMAT (1H1)
    PRINT 400
500 FORMAT (10I8)
    READ 500,NO,MO,NA
  2 FORMAT (3F5.2,F7.0)
    READ 2,US,UO,A,P
  4 FORMAT (I3,I4)
  5 READ 4,N,NN
    SMT=UO
  6 S=N
  7 SN=NN
  8 SUMTA=0.
  9 SUMTS=0.
 10 SQTA=0.
 11 SQTS=0.
 12 TWT=0.
 13 TDT=0.
    TWQT=0.
    TUM=0.
    SQW=0.
    SQD=0.
 14 KK=0
 20 AT(1)=0.
 21 DO 26 I=2,N
 22 CALL SINUO(TA,UO,A,P)
```

```
   TUM=TUM+TA
24 SUMTA=SUMTA+TA
26 AT(I)=SUMTA
27 DO 31 I=1,N
28 CALL EXPNT(TS,US,I)
31 SUM(I)=AT(I)+TS
34 SMIN=SUM(1)
   K=1
35 DO 40 I=2,N
37 IF (SMIN-SUM(I)) 40,40,38
38 SMIN=SUM(I)
39 K=1
40 CONTINUE
41 TEMP=SUM(1)
42 SUM(1)=SMIN
43 SUM(K)=TEMP
45 KK=KK+1
46 IF (KK-NN) 47,47,100
47 CALL SINUO(TA,UO,A,P)
48 SUMTA=SUMTA+TA
49 SQTA=SQTA+TA*TA
50 DIFF=SUMTA-SUM(1)
51 IF (DIFF) 52,55,55
52 WT=-DIFF
   D=0.
53 TWT=TWT+WT
   SQW=SQW+WT*WT
54 GO TO 57
```

```
55 WT=0.
   D=DIFF
56 TDT=TDT+D
   SQD=SQD+D*D
57 CALL EXPNT(TS,US,K)
59 SUMTS=SUMTS+TS
60 SQTS=SQTS+TS*TS
61 SUM(1)=SUMTA+WT+TS
62 GO TO 34
100 TAT=SUMTA-TUM
   AA=TAT/SN
101 CALL VARSD(SDA,TAT,SQTA,SN)
102 AS=SUMTS/SN
103 CALL VARSD(SDS,SUMTS,SQTS,SN)
104 AW=TWT/SN
105 AD=TAT/SN
   DL=SUMTS/(SUMTS+TDT)
   CALL VARSD(SDW,TWT,SQW,SN)
   CALL VARSD(SDD,TDT,SQD,SN)
106 FORMAT (1HK,5X,I4,5X,I3//(1HK,4F20.2))
107 PRINT 106,NN,N,AA,AS,AW,AD,SDA,SDS,SDW,SDD,TWT,
  1TDT,DL
109 GO TO 5
110 END
```

FORTRAN Main Program 5. Multi-Channels With An Ancillary Service Station

```
600 FORMAT (1H1)
    PRINT 600
  1 DIMENSION AT(100),TT1(100)
  2 COMMON NM,M,NO,NL
  3 FORMAT (4I8)
  4 READ 3,NM,M,NO,NL
  5 FORMAT (2F5.2,2I2,3F5.2)
  6 READ 5,UA,UTEST,K1,K2,US,UL,SDL
  7 FORMAT (I3,I4)
  8 READ 7,N,NN
  9 S=N
 10 SN=NN
 12 DO 13 I=1,N
 13 TT1(I)=0.
 14 AT(1)=0.
 15 K=K2
 16 SUMTA=0.
 17 SQTA=0.
 18 WT2=0.
 19 SUMTS=0.
 20 SQTS=0.
 21 SUMTC=0.
 22 SQTC=0.
    SUMTT=0.
    SQTT=0.
 23 TDT2=0.
```

```
 24 TWT2=0.
 25 TDT1=0.
 26 TWT1=0.
 27 LL=0
 28 SUM=0.
 29 TQT1=0.
 30 TQT2=0.
    TQD1=0.
    TQD2=0.
101 DO 105 I=2,N
102 CALL EXPNT(TA,UA)
103 SUM-SUM+TA
104 SUMTA=SUMTA+TA
105 AT(I)=SUMTA
106 CALL GAMMAK(TS,K,US)
107 CALL LGNORM(TC,UL,SDL)
108 TT2=TS
109 TT1(1)=TS+TC
110 DO 123 I=2,N
111 DIF2=AT(I)-TT2
112 IF (DIF2) 115,113,113
113 WT2=0.
    D2=DIF2
114 GO TO 116
115 WT2=-DIF2
    D2=0.
116 IF (WT2-UTEST) 119,119,117
117 K=K1
```

```
118 GO TO 120
119 K=K2
120 CALL GAMMAK(TS,K,US)
121 CALL LGNORM(TC,UL,SDL)
122 TT2=AT(I)+WT2+TS
123 TT1(I)=TT2+TC
201 SMIN=TT1(1)
202 K=1
203 DO 207 I=2,N
204 IF (SMIN-TT1(I)) 207,207,205
205 SMIN=TT1(I)
206 K=I
207 CONTINUE
208 TEMP=TT1(1)
209 TT1(1)=SMIN
210 TT1(K)=TEMP
    IF (LL-NN) 300,400,400
300 CALL EXPNT(TA,UA)
    LL=LL+1
301 SUMTA=SUMTA+TA
302 SQTA=SQTA+TA*TA
303 DIF1=SUMTA-TT1(1)
    IF (DIF1) 307,304,304
304 WT1=0.
    D1=DIF1
305 TDT1=TDT1+D1
    TQD1=TQD1+D1*D1
306 GO TO 309
```

```
307 WT1=-DIF1
    D1=0.
308 TWT1=TWT1+WT1
    TQT1=TQT1+WT1*WT1
309 DIF2=SUMTA-TT2
310 IF (DIF2) 314,311,311
311 WT2=0.
    D2=DIF2
312 TDT2=TDT2+D2
    TQD2=TQD2+D2*D2
313 GO TO 316
314 WT2=-DIF2
    D2=0.
315 TWT2=TWT2+WT2
    TQT2=TQT2+WT2*WT2
316 IF (WT2-UTEST) 319,319,317
317 K=K1
318 GO TO 320
319 K=K2
320 CALL GAMMAK(TS,K,US)
321 CALL LGNORM(TC,UL,SDL)
501 FORMAT (1HK,I6,7F15.2)
    PRINT 501,LL,TA,WT1,D1,WT2,D2,TS,TC
    SUMTS=SUMTS+TS
    SQTS=SQTS+TS*TS
    SUMTC=SUMTC+TC
    SQTC=SQTC+TC*TC
    SUMTT=SUMTT+WT2+TS+TC
```

```
      SQTT=SQTT+(WT2+TS+TC)*(WT2+TS+TC)
  322 TT2=SUMTA+WT1+WT2+TS
  323 TT1(1)=TT2+TC
  324 GO TO 201
  400 TAT=SUMTA-SUM
  401 AAT=TAT/SN
  402 AST=SUMTS/SN
  403 ACT=SUMTC/SN
  404 AWT1=TWT1/SN
  405 AWT2=TWT2/SN
  406 AID1=TDT1/SN
  407 AID2=TDT2/SN
  408 ATS=AWT1+AWT2+AST+ACT
      DL1=(SUMTS+SUMTC+TWT2)/(SUMTS+SUMTC+TWT2+TDT1)
      DL2=SUMTS/(SUMTS+TDT2)
      CALL VARSD(SDA,TAT,SQTA,SN)
      CALL VARSD(SDS,SUMTS,SQTS,SN)
      CALL VARSD(SDC,SUMTC,SQTC,SN)
      CALL VARSD(SDW1,TWT1,TQT1,SN)
      CALL VARSD(SDW2,TWT2,TQT2,SN)
      CALL VARSD(SDD1,TDT1,TQD1,SN)
      CALL VARSD(SDD2,TDT2,TQD2,SN)
      CALL VARSD(SDT,SUMTT,SQTT,SN)
  409 PRINT 410,N,NN,AAT,AWT1,AID1,AWT2,AST,AID2,ACT,
     1ATS,SDA,SDW1,SDD1,SDW2,SDS,SDD2,SDC,SDT,TWT1,
     2TWT2,TDT1,TDT2,DL1,DL2
  410 FORMAT (1HK,2I15/(1HK,8F15.2))
  413 END
```

FORTRAN Main Program 6. Multi-Channels, Two Intervals of Service Time

```
  1 DIMENSION AT(100),SUM(100)
    COMMON NO,MO,NG,NL,NR
400 FORMAT (1H1)
    PRINT 400
500 FORMAT (5I8)
    READ 500,NO,MO,NG,NL,NR
  2 FORMAT (F5.2,F2.2,2I2,6F5.2)
  3 READ 2,U,P,KD1,KD2,US,TEST,UL1,VL1,UL2,VL2
  4 FORMAT (I3,I4)
  5 READ 4,N,NN
  6 S=N
  7 SN=NN
  8 SUMTA=0.
  9 SUMTS=0.
    SUMTD =0.
    SUMTL=0.
    SQTD=0.
    SQTL=0.
 10 SQTA=0.
 11 SQTS=0.
 12 TWT=0.
 13 TST-0.
    TWQT=0.
    TUM=0.
    SQW=0.
    SQD=0.
```

```
 14 KK=0
 20 AT(1)=0.
 21 DO 26 I=2,N
 22 CALL EXPNT(TA,U)
    TUM=TUM+TA
 24 SUMTA=SUMTA+TA
 26 AT(I)=SUMTA
 27 DO 31 I=1,N
600 CALL RANDOM(R)
601 IF (R-P) 602,604,604
602 KD=KD1
603 GO TO 605
604 KD=KD2
605 CALL GAMMAK(TD,KD,US)
606 IF (TD-TEST) 607,609,609
607 UL=UL1
    VL=VL1
608 GO TO 610
609 UL=UL2
    VL=VL2
610 CALL LGNORM(TL,UL,VL)
611 TS=TD+TL
 31 SUM(I)=AT(I)+TS
 34 SMIN=SUM(1)
 35 K=1
 36 DO 40 I=2,N
 37 IF (SMIN-SUM(I)) 40,40,38
 38 SMIN=SUM(I)
```

```
39 K=I
40 CONTINUE
41 TEMP=SUM(1)
42 SUM(1)=SMIN
43 SUM(K)=TEMP
45 KK=KK+1
46 IF (KK-NN) 47,47,100
47 CALL EXPNT(TA,U)
48 SUMTA=SUMTA+TA
49 SQTA=SQTA+TA*TA
50 DIFF=SUMTA-SUM(1)
51 IF (DIFF) 52,55,55
52 WT=-DIFF
   D=0.
53 TWT=TWT+WT
   SQW=SQW+WT*WT
54 GO TO 700
55 WT=0.
   D=DIFF
56 TDT=TDT+D
   SQD=SQD+D*D
700 CALL RANDOM(R)
701 IF (R-P) 702,704,704
702 KD=KD1
703 GO TO 705
704 KD=KD2
705 CALL GAMMAK(TD,KD,US)
706 IF (TD-TEST) 707,709,709
```

```
707 UL=UL1
    VL=VL1
708 GO TO 710
709 UL-UL2
    VL=VL2
710 CALL LGNORM(TL,UL,VL)
711 TS=TD+TL
200 FORMAT (1HK,I6,5F15.2)
    PRINT 200,KK,TA,WT,D,TD,TL
 59 SUMTS=SUMTS+TS
 60 SQTS=SQTS+TS*TS
    SUMTD=SUMTD+TD
    SUMTL=SUMTL+TL
    SQTD=SQTD+TD*TD
    SQTL=SQTL+TL*TL
 61 SUM(1)=SUMTA+WT+TS
 62 GO TO 34
100 TAT=SUMTA-TUM
    AA=TAT/SN
101 CALL VARSD(SDA,TAT,SQTA,SN)
    CALL VARSD(SDTD,SUMTD,SQTD,SN)
102 AS=SUMTS/SN
103 CALL VARSD(SDS,SUMTS,SQTS,SN)
    CALL VARSD(SDTL,SUMTL,SQTL,SN)
104 AW=TWT/SN
    ATD=SUMTD/SN
    ATL=SUMTL/SN
105 AD=TOT/SN
    CALL VARSD(SDW,TWT,SQW,SN)
```

```
      CALL VARSD(SDD,TDT,SQD,SN)
      DL=SUMTS/(SUMTS+TDT)
  106 FORMAT (1HK,5X,I4,5X,I3//(1HK,6F20.2))
  107 PRINT 106,NN,N,AA,ATD,ATL,AS,AW,AD,SDA,SDTD,SDTL,
     1SDS,SDW,SDD,TWT,TDT,DL
  109 GO TO 5
  110 END
```

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